Online Appendix

to "The Short-Run Employment Effects of Public Infrastructure Investment"

A Calibration Details

A.1 Estimation of job-finding and separation probabilities

The data source most commonly used to estimate transition rates between labor market states is the Current Population Survey (CPS). There are two main method to estimating the job-finding rate from CPS data. Here, I use the one based on gross flows, that is, I use the panel dimension of the monthly CPS microdata to estimate the number of workers who transition from unemployment to employment in a given month. The alternative approach uses only the aggregate time series of unemployment as described in Shimer (2012). It requires stronger assumptions than the gross flows method used here, in particular, it assumes a constant labor force. In contrast, the gross flows approach can be extended to incorporate more than two labor market states and arbitrary transitions between them. A discussion and comparison of the two methods can be found in Shimer (2012).

I consider two different definitions of unemployed workers, denoted U-3 and U-5 by the BLS. The most widely used concept is U-3. According to this definition a worker is unemployed if i) he or she does not work but has been actively looking for a job during the last four weeks and would be available to work or if ii) he or she is temporarily laid off and waiting to be recalled. The alternative definition, U-5, also encompasses workers who want a job, searched for a job at some point during the last twelve months, and could have taken a job in the last week if they had been offered one. Hence, this measure includes discouraged and marginally attached workers according to the BLS classification. Figure 11 shows the number of unemployed workers according to the definitions U-3 and U-5 over time.

Following Shimer (2012), I estimate the job-finding probability from gross flows as follows:

- I match individuals across monthly CPS waves from January 1976 to December 2020 to obtain a panel data set
- For every month I compute the number of workers who transition between each of the three labor market states employed, unemployed, inactive
  - I do this for both concepts of unemployment, U-3 and U-5
  - The series are seasonally adjusted using X13-ARIMA-SEATS

- From these flows I obtain a Markov matrix for the monthly transition between the three states for every month in the sample
- I adjust for time aggregation using the method described in Shimer (2012)
  - I compute the continuous time Markov matrix (instantaneous transition probabilities) from the discrete time matrix and obtain the monthly transition probabilities from the instantaneous transition rates. The monthly probabilities obtained in this way capture the probability of experiencing a transition between state A and B over the course of one month. This is different from the probability of being in state B in the next month conditional on being in state A in the current month. The latter is what I observe in the data, the former is what I need to inform the calibration of the model.
- To also obtain separate transition probabilities for U-3 unemployed and marginally attached workers, I use the same procedure but with four states (employed, U-3 unemployed, marginally attached, inactive).

Prior to 1994, the CPS did not include the questions used to identify discouraged and marginally attached workers. This is why I can only compute job-finding probabilities of unemployed workers according to the broader definition U-5 for the time period from 1994 to 2020. For comparison, I also compute the transition probabilities according to the unemployment concept U-3 for the whole time period covered by the CPS, January 1976 to December 2020. Table 6 shows the average monthly job-finding probability for U-3 unemployed, U-5 unemployed, and marginally attached workers for different time periods. For the time period from 1994 to 2020, the average job-finding probability for unemployed workers according to the concept U-3 was 29.4%. It was 2.5 percentage points lower for the group of U-5 unemployed workers. Marginally attached workers are much less likely to find a job in a given month, on average their job-finding probability is only 10.9%.

Table 6: Average monthly transition probabilities, 1976–2020 and 1994–2020.

	1976–2020	1994–2020
Job-finding probability U-3	29.8	29.4
Job-finding probability U-5	—	26.9
Job-finding probability marginally attached	—	10.9
Separation rate	1.9	1.8

The reason for the small difference in job-finding probabilities between U-3 and U-5 can be found in Figure 11, which shows the total numbers of unemployed workers according to definitions U-3 and U-5 and the number of marginally attached workers over time. On average, the number of marginally attached workers is only about one fifth of the number of U-3 unemployed workers. For the group of unemployed workers according to the definition U-5, marginally attached workers play a small role. This is why the substantially lower job-finding probability of marginally attached workers does not matter much for the overall job-finding probability in the group of U-5 unemployed workers.

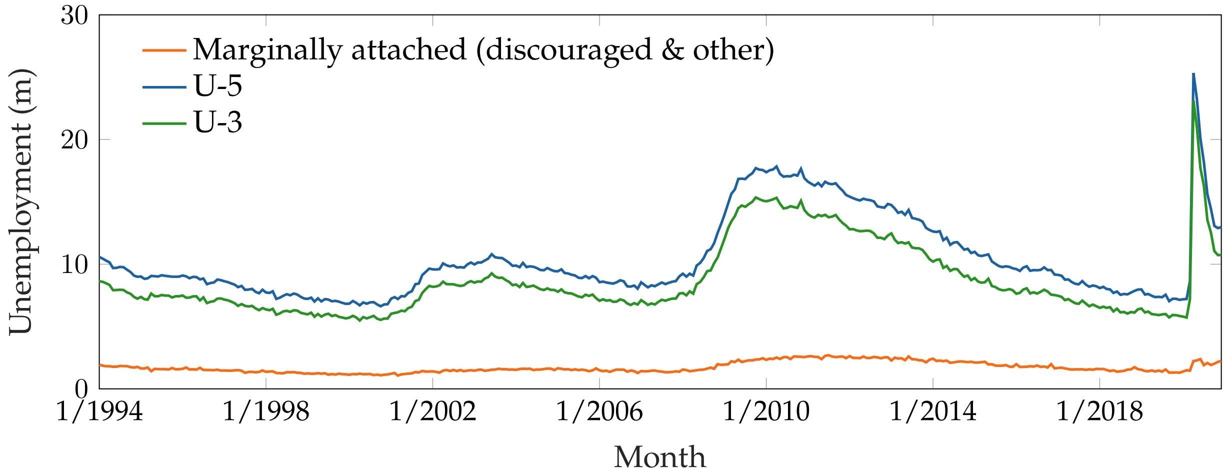

Figure 11: Unemployment in the US, January 1994 to December 2020 (in millions).

Figure 12 shows the estimated monthly job-finding probability over time. The dark blue line shows the estimated monthly job-finding probability of unemployed workers, when unemployed according to the concept U-3 are considered. For the time period from 1976Q1 to 2007Q2, I can compare the quarterly averages of this series to the series in Shimer (2012). The two are very similar, the standard deviation of the difference is less than 1.5 percentage points. This difference is likely coming from the different seasonal adjustment procedures used. The light blue line represents the job-finding probability for unemployed according to the definition U-5. Finally, the green line shows the job-finding rate for marginally employed workers, when I distinguish between four labor market states, employed, U-3 unemployed, marginally attached, and inactive.

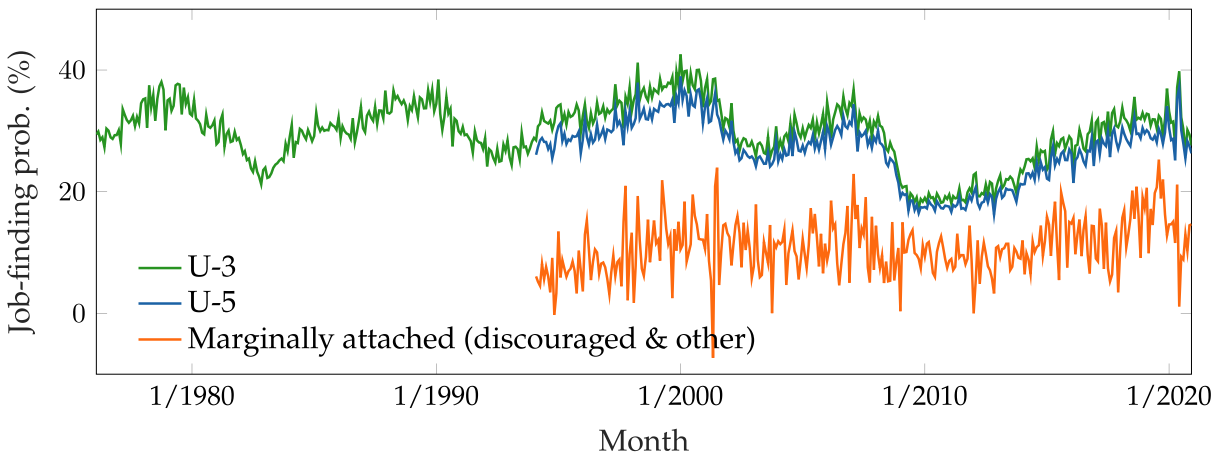

Figure 12: Estimated monthly job-finding probabilities.

A.2 Calibration of disutility from effort

I calibrate the parameter χ to match the elasticity of the job-finding probability with respect to unemployment benefits $\epsilon_{\pi,b} = \frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{b}{\pi^{e|u}}$. From the first-order condition for search effort, I have that

$$\ell^{\chi} = \beta\left(J_t(e) - J_t(u)\right)\frac{\pi^{e|u}}{\ell}. \tag{33}$$

In the steady state the difference between lifetime utility of employed and unemployed workers is

$$J_t(e) - J_t(u) = \frac{\log\left(\frac{w}{b}\right) - d_{0,e} + \frac{\ell^{1+\chi}}{1+\chi}}{1-\beta+\beta(\rho+\pi^{e|u})}$$

Hence

$$\left(1-\beta+\beta(\rho+\pi^{e|u})\right)\ell^{\chi} = \beta\left(\log\left(\frac{w}{b}\right) - d_{0,e} + \frac{\ell^{1+\chi}}{1+\chi}\right)x$$

where $x = \frac{\pi^{e|u}}{\ell}$ is a constant (partial equilibrium) and

$$\left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi-1}\frac{\mathrm{d}\ell}{\mathrm{d}b} + \beta\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\ell^{\chi} = -\beta\frac{1}{b}x + \beta\ell^{\chi}x\frac{\mathrm{d}\ell}{\mathrm{d}x}$$

Since, $\frac{\mathrm{d}\ell}{\mathrm{d}b} = \frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{1}{x}$.

$$\left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi-1}\frac{1}{x}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b} + \beta\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\ell^{\chi} = -\frac{\beta}{b}x + \beta\ell^{\chi}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b} \tag{34}$$

$$\Leftrightarrow \left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{1}{\pi^{e|u}} = -\beta\frac{1}{b}x \tag{35}$$

$$\Leftrightarrow \left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{b}{\pi^{e|u}} = -\beta\frac{\pi^{e|u}}{\ell} \tag{36}$$

Substituting (33) for ℓ^{χ} and rearranging yields

$$\chi = -\frac{1}{(1-\beta+\beta(\rho+\pi^{e|u}))\epsilon_{q,b}\left(J_t(e) - J_t(u)\right)}$$

All terms on the right-hand side follow directly from the calibration targets.

B Additional Theoretical Results

B.1 Welfare effects of public investment

The permanent expansion in public investment raises employment as firms expand hiring in anticipation of higher future productivity. I now show that this increase in employment constitutes an efficiency gain when equilibrium labor demand is inefficiently low. In this case, public investment improves labor market efficiency because the anticipation effect stimulates labor demand and brings vacancy creation closer to its efficient level. Therefore, public investment has a positive effect on welfare beyond the return from public investment and redistribution.

I define social welfare as follows

$$\begin{aligned} W(\{c_t^F, c_t(s^t), \ell_t(s^t)\}) =& \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t u^F(c_t^F) \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u\left(c_t(s^t), \ell_t(s^t), s_t\right) \pi_t(s^t|s_0) \bar{\mu}(s_0). \end{aligned}$$

Here, $\bar{\mu}^F$, $\bar{\mu}(e)$ and $\bar{\mu}(u)$ are the welfare weights of firm owners, initially employed and initially unemployed workers and $\pi_t(s^t|s_0)$ denotes the share of workers with history $s^t = (s_0, s_1, \ldots, s_t)$ in period t. Let C_t denote aggregate consumption in period t and define the consumption shares of individual firm owners and of workers as $v_t^F \equiv \frac{c_t^F}{C_t}$ and $v_t(s^t) \equiv \frac{c_t(s^t)}{C_t}$.

Under Assumption 1 (fixed search effort), the effect of the investment program on welfare is

$$\begin{aligned} \frac{\partial W}{\partial x} =& \underbrace{\sum_{t=0}^{\infty} \beta^t C_t \left(\bar{\mu}^F u_c^F(c_t^F) \frac{\partial v_t^F}{\partial x} + \sum_{s^t} \bar{\mu}(s_0) \pi_t(s^t|s_0) u_c(c_t(s^t)) \frac{\partial v_t(s^t)}{\partial x} \right)}_{\text{redistribution (intensive margin)}} \\ &+ \underbrace{\sum_{t=0}^{\infty} \beta^t u(c_t(s^t), \ell_t(s^t), s_t) \bar{\mu}(s_0) \frac{\partial \pi_t(s^t|s_0)}{\partial x}}_{\text{redistribution (extensive margin)}} + \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x}}_{\text{aggregate consumption}}. \end{aligned} \tag{37}$$

Here,

$$\mathrm{m}_t \equiv \bar{\mu}^F v_t^F u_c^F(c_t^F) + \sum_{s^t} \bar{\mu}(s_0) \pi_t(s^t|s_0) v_t(s^t) u_c(c_t(s^t)) \tag{38}$$

is the marginal utility of aggregate consumption in period t, a weighted average of individual marginal utilities of consumption, where the weight of each agent corresponds to its welfare weight multiplied by its consumption share.

As can be seen from equation (37), the effect of the expansion in public investment on welfare can be decomposed into three parts. The first captures the effect of public investment on the distribution of consumption along the intensive margin. Depending on how the increase in public investment is financed, consumption of employed workers, unemployed workers or firm owners increases or falls relative to aggregate consumption and this redistribution changes welfare, even if aggregate consumption remains unchanged. This distributive effect is captured by the first line in equation (37). Note that under Assumption 2 wages are independent of taxes such that the government can use labor taxes and lump-sum taxes on firm owners to finance investment in a way that leaves the consumption shares of all households unchanged. In this case there is no redistribution of consumption along the intensive margin and the first line in (37) is zero.

The second effect on welfare emerges because the increase in public investment redistributes consumption (and effort) along the extensive margin as it alters the share of workers who are employed. Proposition 1 showed that employment increases in all periods in response to a permanent expansion in public investment if the wage and public investment are in steady state initially. Hence, the extensive margin redistribution raises welfare for sensible parameter choices under which the after-tax wage exceeds unemployment benefits and compensates for potential utility losses from working.

The last summand in equation (37) captures the welfare effect of changes in aggregate consumption due to a permanent increase in public investment. The change in aggregate consumption is

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t K_t^{\alpha} N_t^{1-\alpha} \frac{\partial z_t}{\partial x}}_{\text{direct gross return}} - \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial I_t^G}{\partial x}}_{\text{costs}} + \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t}_{\text{efficiency gain}} \tag{39}$$

Equation (39) shows that there are three channels through which the permanent increase in public investment affects aggregate consumption. The first two are standard. On the one hand, public investment raises productivity, which leads to an increase in output and consumption. On the other hand, there is a resource cost of public investment that reduces consumption. In the frictional labor market considered here, there is a third channel through which public investment affects output. I label it EG_t for "Efficiency Gain" in equation (39).

If the economy is in the steady state, the efficiency gain is

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t = \frac{1}{1-\eta} \left[w - \eta \left((1-\alpha) z k^{\alpha} + \theta \kappa \right) \right] \sum_{t=0}^{\infty} \beta^t m_t M_{t+1}^{Inv}.$$

It comes from the fact that the equilibrium in the matching labor market is not necessarily efficient such that the employment effect of public investment by itself can improve welfare.[16] When a firm posts a vacancy, it imposes a negative externality on other firms, since the additional vacancy makes it more difficult for other firms to fill theirs. However, there is also a positive externality because every additional vacancy makes it easier for workers to find a job. As shown by Hosios (1990), there exists a wage that internalizes both effects and leads to the optimal level of vacancy creation. This wage is such that workers' share of the total match surplus equals the elasticity of the matching function. Here, this is the case if

$$w^* = \eta \left((1-\alpha) z k^{\alpha} + \theta \kappa \right).$$

16. For simplicity, I assume that vacancy posting costs are constant, $\kappa_t = \kappa$. Below, I characterize the effect of public investment on aggregate consumption for the general case in which posting costs can depend on public investment.

I show this formally in Appendix B.2, where I derive the constrained efficient allocation. When the equilibrium wage equals the efficient wage, $w = w^*$, the efficiency gain is zero. In contrast, if the wage exceeds the efficient wage, $w > w^*$, vacancy creation in equilibrium is too low and the expansion in labor demand brought about by the investment program can raise the amount of resources available for consumption. The following proposition summarizes this result.

Proposition 5 (Efficiency gains from public investment)**.** *Suppose the economy is in a steady state with inefficiently low labor demand, $w > w^*$. Then, the public investment program improves labor market efficiency,*

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t > 0.$$

Proof. The welfare function is

$$\begin{aligned} W(\{c_t^F, c_t(s^t), \ell_t(s^t)\}) =& \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t u^F(c_t^F) \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u\left(c_t(s^t), \ell_t(s^t), s_t\right) \pi_t(s^t|s_0)\bar{\mu}(s_0). \end{aligned}$$

We can equivalently express welfare as a function of aggregate consumption and individual consumption shares

$$\begin{aligned} \tilde{W}(\{v_t^F, v_t(s^t), \ell_t(s^t), C_t\}) =& \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t u^F(v_t^F C_t) \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u\left(v_t(s^t)C_t, \ell_t(s^t), s_t\right) \pi_t(s^t|s_0)\bar{\mu}(s_0), \end{aligned}$$

such that

$$\begin{aligned} \frac{\partial W}{\partial x} = \frac{\partial \tilde{W}}{\partial x} =& \sum_{t=0}^{\infty} \beta^t \bar{\mu}^F u_c^F(c_t^F) C_t \frac{\partial v_t^F}{\partial x} \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u_c(c_t(s^t)) C_t \frac{\partial v_t(s^t)}{\partial x} \pi_t(s^t|s_0)\bar{\mu}_0(s_0) \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u(c_t(s^t), \ell_t(s^t), s_t) \frac{\pi_t(s^t|s_0)}{\partial x} \bar{\mu}_0(s_0) \\ &+ \sum_{t=0}^{\infty} \beta^t v_t^F u_c^F(c_t^F) \bar{\mu}^F \frac{\partial C_t}{\partial x} \\ &+ \sum_{t=0}^{\infty} \sum_{s^t} \beta^t v_t(s^t) u_c(c_t(s^t)) \pi_t(s^t|s_0) \bar{\mu}(s_0) \frac{\partial C_t}{\partial x}, \end{aligned}$$

which yields (37). Furthermore,

$$C_t = z_t N_t^{1-\alpha} K_t^{\alpha} - \kappa_t \theta_t (1 - N_t) - K_{t+1} + (1-\delta_k) K_t - I_t^G,$$

such that

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = &\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \Bigg(N_t^{1-\alpha} K_t^{\alpha} \frac{\partial z_t}{\partial x} + (1-\alpha) z_t N_t^{-\alpha} K_t^{\alpha} \frac{\partial N_t}{\partial x} \\
&+ \alpha z_t N_t^{1-\alpha} K_t^{\alpha-1} \frac{\partial K_t}{\partial x} \\
&+ \kappa_t \theta_t \frac{\partial N_t}{\partial x} - \left(\frac{\partial \kappa_t}{\partial x} \theta_t + \kappa_t \frac{\partial \theta_t}{\partial x} \right) (1 - N_t) \\
&- \frac{\partial K_{t+1}}{\partial x} + (1-\delta_k) \frac{\partial K_t}{\partial x} - \frac{\partial I_t^G}{\partial x} \Bigg) \\
= &\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \left(N_t^{1-\alpha} K_t^{\alpha} \frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial x} \right) \\
&+ \sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \Bigg(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k \right) \frac{\partial K_t}{\partial x} \\
&- \frac{\partial K_{t+1}}{\partial x} - \theta_t (1 - N_t) \frac{\partial \kappa_t}{\partial x} \Bigg) \\
&+ \sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \left(\left[(1-\alpha) z_t k_t^{\alpha-1} + \kappa_t \theta_t \right] \frac{\partial N_t}{\partial x} - \kappa_t (1 - N_t) \frac{\partial \theta_t}{\partial x} \right)
\end{aligned}$$

From the law of motion for employment, we get

$$\kappa_t (1 - N_t) \frac{\partial \theta_t}{\partial x} = \left[\frac{\partial N_{t+1}}{\partial x} - (1 - \rho - q_t^v(\theta_t) \theta_t) \frac{\partial N_t}{\partial x} \right] \frac{\kappa_t}{(1-\eta) q_t^v(\theta_t)}$$

Using this, we have

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = &\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\left(N_t^{1-\alpha}K_t^{\alpha}\frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial .x}\right)\\
&+\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\Bigg(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right)\frac{\partial K_t}{\partial x}\\
&-\frac{\partial K_{t+1}}{\partial x} - \theta_t(1-N_t)\frac{\partial \kappa_t}{\partial x}\Bigg)\\
&+\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\Bigg(\Big[(1-\alpha)z_t k_t^{\alpha-1}\\
&+\kappa_t\theta_t\left(1+\frac{1-\rho-q_t^v(\theta_t)}{(1-\eta)q_t^v(\theta_t)}\right)\Big]\frac{\partial N_t}{\partial x}\\
&-\frac{\kappa_t}{(1-\eta)q_t^v(\theta_t)}\frac{\partial N_{t+1}}{\partial x}\Bigg)
\end{aligned}
$$

and with the equilibrium condition

$$
\frac{\kappa_t}{q_t^v(\theta_t)} = \beta\left\{(1-\alpha)z_{t+1}k_{t+1}^{\alpha} - w_{t+1} + (1-\rho)\frac{\kappa_{t+1}}{q_t^v(\theta_{t+1})}\right\}
$$

we get

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = &\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\left(N_t^{1-\alpha}K_t^{\alpha}\frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial .x}\right)\\
&+\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\Bigg(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right)\frac{\partial K_t}{\partial x}\\
&-\frac{\partial K_{t+1}}{\partial x} + \theta_t(1-N_t)\frac{\partial \kappa_t}{\partial x}\Bigg)\\
&+\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\left(\left[(1-\alpha)k_t^{\alpha}z_t + \kappa_t\theta_t\right]\frac{\partial N_t}{\partial x}\right)\\
&-\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\frac{\kappa_t\theta_t}{1-\eta}\frac{\partial N_t}{\partial x}\\
&-\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\beta\Bigg(\frac{(1-\alpha)k_{t+1}^{\alpha}z_{t+1} - w_{t+1}}{1-\eta}\\
&+\frac{(1-\rho)\kappa_{t+1}}{(1-\eta)q_t^v(\theta_{t+1})}\Bigg)\frac{\partial N_{t+1}}{\partial x}\\
&+\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\frac{(1-\rho)\kappa_t}{(1-\eta)q_t^v(\theta_t)}\frac{\partial N_t}{\partial x}
\end{aligned} \tag{40}
$$

Suppose the economy is in a steady state, then the average marginal utility of consumption $\mathrm{m}_t = v^F \bar{\mu}_0 + \frac{1}{C}\sum_{s_0} \bar{\mu}(s_0)$. Then, since $\frac{\partial N_0}{\partial x} = 0$,

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{(1-\rho)\kappa_t}{(1-\eta)q_t^v(\theta_t)} \frac{\partial N_t}{\partial x} = \sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{(1-\rho)\kappa_t}{(1-\eta)q_{t+2}^v(\theta_{t+1})} \frac{\partial N_{t+1}}{\partial x}$$

and the two terms cancel in equation (40). Furthermore, it follows from the optimal capital choice (see (16)) that $\alpha z_t k_t^{\alpha-1} + 1 - \delta_k = \frac{1}{\beta}$. Together with $\frac{\partial K_0}{\partial x} = 0$ this implies

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \left(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right) \frac{\partial K_t}{\partial x} - \frac{\partial K_{t+1}}{\partial x} \right) = 0,$$

which simplifies equation (40) further and yields

$$\sum_{t=0}^{\infty} \beta^t m_t \frac{\partial C_t}{\partial x} = \sum_{t=0}^{\infty} \beta^t m_t \left(N^{1-\alpha} K^{\alpha} \frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial x} \right) - \sum_{t=0}^{\infty} \beta^t m_t \theta (1-N) \frac{\partial \kappa_t}{\partial x}$$
$$+ \sum_{t=0}^{\infty} \beta^t m_t \frac{1}{1-\eta} \left[w - \eta \left((1-\alpha) z k^{\alpha} + \theta \kappa \right) \right] M_{t+1}^{Inv}.$$

The second term in the first line are the costs (or benefits) of changing vacancy posting costs. If posting costs are constant, the term drops out. The second line is the efficiency gain as defined in the main text. Since, the employment multiplier $M_{t+1}^{Inv} > 0$ is positive (proposition 1), the efficiency gain is positive if

$$w > \eta \left((1-\alpha) z k^{\alpha} + \theta \kappa \right) = w^*.$$

□

A similar result can be found in the online appendix of Den Haan and Kaltenbrunner (2009) who study a simplified two-period model. They find that news about future productivity can lead to a resource gain when the Hosios condition is violated.

B.2 Optimal allocation

In general, the equilibrium in the search and matching labor market described above is inefficient due to two congestion externalities. When posting a vacancy, a firm does not take into account the negative effect this has on the likelihood of other firms to fill their vacancies. Similarly, firms fail to internalize that every additional vacancy makes it easier for workers to find a job. As a result, the private benefits of posting a vacancy may exceed or fall below the social benefit.

To better understand how these inefficiencies shape the effects of government investment, I analyze the constrained-efficient allocation, which I define as the one that would be chosen by a utilitarian social planner who is constrained by the matching friction and faces the same capital adjustment costs as firm owners. To that end, I define social welfare as

$$
\begin{aligned}
W(\{c_t^F, c_t(s^t)\}) =& \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t c_t^F + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \log\left(c_t(s^t)\right) \\
& - d(\ell_t(s^t))\pi_t(s^t|s_0)\bar{\mu}(s_0),
\end{aligned}
$$

where $\bar{\mu}^F$, $\bar{\mu}(e)$ and $\bar{\mu}(u)$ are the welfare weights of firm owners, initially employed and initially unemployed workers and $\pi_t(s^t|s_0)$ denotes the share of workers with history $s^t = (s_0, s_1, \ldots, s_t)$ in period t.

Definition 3 (Optimal allocation)**.** *An optimal allocation for a given sequence of productivity is a collection of sequences of aggregate consumption, capital, employment, search effort and labor market tightness and of individual consumption and search effort which solves the planner problem*

$$
\begin{aligned}
\max_{\{C_t, N_{t+1}, K_{t+1}, L_t^u, \theta_t, c_t^F, c_t(s^t), \ell_t(s^t)\}} \quad & W(\{c_t^F, c_t(s^t)\}) \\
s.t.\ & C_t + K_{t+1} + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t + \kappa_t \theta_t L_t^u \\
& \quad = z_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta_k)K_t \\
& N_{t+1} = (1-\rho)N_t + q_t^v(\theta_t)\theta_t L_t^u \qquad (41) \\
& C_t = \mu c_t^F + \sum_{s^t} c_t(s^t)\pi_t(s^t) \\
& L_t^u = \sum_{s^t|s_t=u} \ell_t(s^t)\pi_t(s^t) \\
& \textit{given } K_0,\ N_0.
\end{aligned}
$$

The planner takes the sequence of productivity as given. In other words, the sequence of public investment and thereby productivity has already been decided, and the planner now faces the problem of allocating the remaining resources.[17] The first constraint in the planner problem is the aggregate resource constraint. The right-hand side are total available resources consisting of output and capital after depreciation which can be spent on consumption, investment in next period's capital, and vacancy creation. The second constraint is the law of motion for employment. The planner can increase employment in the next period in two ways. First, the planner can raise tightness θ_t which comes at a resource cost according to the term $\kappa_t \theta_t L_t^u$ in the resource constraint since more vacancies have to be created for a constant level of aggregate search effort. Second, employment can be increased by raising aggregate search effort L_t^u with comes at a utility cost since effort enters the utility function, but there are also resource costs since more vacancies have to be

17. The costs of public investment could be added to the resource constraint without changing the results that follow. This is because firm owners have linear utility.

created if tightness is to be held constant. The last two constraints of the planner problem state that individual consumption must add up to aggregate consumption and individual search effort $\ell_t(s^t)$ has to be consistent with aggregate search effort L_t^u.

The next propositions characterize the optimal allocation more closely.

Proposition 6 (Optimal allocation of capital)**.** *The optimal allocation of capital satisfies*

$$\begin{aligned}1+&\phi\left(\frac{K_{t+1}}{K_t}-1\right)\\&=\beta\left(1+\alpha z_{t+1}k_{t+1}^{\alpha-1}-\delta_k+\frac{\phi}{2}\left(\left(\frac{K_{t+2}}{K_{t+1}}\right)^2-1\right)-\frac{\partial\kappa_{t+1}}{\partial K_{t+1}}\theta_{t+1}L_{t+1}^u\right).\end{aligned}$$

Proof. The result follows immediately from the first-order conditions for consumption and capital associated with (41). □

If vacancy posting costs do not depend on capital, it holds that $\frac{\partial\kappa_{t+1}}{\partial K_{t+1}}=0$ and the optimal path for the aggregate capital stock coincides with the equilibrium allocation. However, if vacancy posting costs depend on the aggregate capital stock, for example because they are proportional to labor productivity as would be needed for balanced growth, then the aggregate capital stock is too high in equilibrium because existing firms who rent capital do not take into account that more capital per match makes it more expensive for new firms to post a vacancy.

Next, I characterize the sequence of optimal tightness. It will depend on the elasticity of the vacancy filling probability with respect to tightness which I denote as $\eta\equiv-\frac{m'(\theta_t)\theta_t}{q_t^v(\theta_t)}$.

Proposition 7 (Optimal tightness with fixed search effort)**.** *Suppose individual search effort is fixed at $\ell_t(s^t)=1$ and $d(1,u)=d(1,e)$, then optimal tightness satisfies*

$$\begin{aligned}\frac{\kappa_t}{q_t^v(\theta_t)}=&\beta\Bigg\{(1-\alpha)z_{t+1}k_{t+1}^\alpha-\eta\left[(1-\alpha)z_{t+1}k_{t+1}^\alpha+\kappa_{t+1}\theta_{t+1}\right]\\&+(1-\rho)\frac{\kappa_{t+1}}{q_{t+1}^v(\theta_{t+1})}\Bigg\}.\end{aligned}$$

Comparison with the equilibrium condition (7) shows that without search effort, the equilibrium is constrained efficient if the wage is

$$w_t=\eta\left[(1-\alpha)z_tk_t^\alpha+\kappa_t\theta_t\right] \tag{42}$$

This is the standard condition for efficiency in the DMP model.

Proposition 8 (Optimal tightness)**.** *Suppose that the welfare weights of initially unemployed and employed workers are equal to their population shares, $\bar{\mu}(s)=\pi_0(s)$, then optimal tightness* *satisfies*

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta\Bigg\{(1-\alpha)z_{t+1}k_{t+1}^{\alpha} - \eta\left[(1-\alpha)z_{t+1}k_{t+1}^{\alpha} + \kappa_{t+1}\theta_{t+1}\ell_{t+1}(u)\right]$$
$$+ (1-\eta)\frac{\mu}{\bar{\mu}^F}\left(d(\ell_{t+1}(u),u) - d(0,e)\right) + (1-\rho)\frac{\kappa_{t+1}}{q_{t+1}^v(\theta_{t+1})}\Bigg\},$$

where the optimal level of individual search effort solves

$$d'(\ell_t(u),u) = \frac{\bar{\mu}^F}{\mu}\kappa_t\theta_t\frac{1}{1+\eta}.$$

In this case, the constrained-efficient allocation is implemented if the wage amounts to

$$w_t = \eta\left[(1-\alpha)z_t k_t^{\alpha} + \kappa_t\theta_t\ell_t(u)\right] - (1-\eta)\frac{\mu}{\bar{\mu}^F}\left(d(\ell_t(u),u) - d(0,e)\right). \tag{43}$$

The differences to the optimal wage in the case without effort given by equation (42) are intuitive. First, the term $\kappa_t\theta_t$ is multiplied by individual search effort $\ell_t(u)$. To see why, suppose optimal search effort increases. Then, firms find it easier to fill a vacancy and expand vacancy creation. To prevent an inefficiently high vacancy creation, the wage has to be higher to discourage vacancy creation. Second, the additional summand in (43) takes into account the difference in disutility of effort between employed and unemployed. If the disutility is higher for unemployed, a lower level of unemployment is desirable which is implemented through a lower wage leading to a higher level of labor market tightness.

B.3 No-trade equilibrium and interest rate

I want to show that hand-to-mouth behavior can be the equilibrium outcome in an extended model in which households can save in a risk-free bond a_t at rate r_t, but are borrowing constrained. Consider the following generalization of the household problems described in the main text. Workers are excluded from participation in the equity and capital market where firm owners trade shares and rent out capital. A worker's budget constraint is

$$c_t(s_t) \leq (1-\tau_t)w_t\mathbb{1}_{s_t=e} + b_t\mathbb{1}_{s_t=u}(1+r_t)a_t - a_{t+1} \tag{44}$$

with borrowing constraint $a_{t+1} \leq 0$. Workers choose private consumption, effort, and bond holdings $\{c_t(s_t), \ell_t(s_t), a_{t+1}(s_t)\}_{t=0}^{\infty}$ to maximize expected lifetime utility subject to the budget constraint (44) and the borrowing limit $a_{t+1} \geq 0$. Observe that the wage depends on asset holdings a_t since it is determined by Nash bargaining and worker are risk averse such that their surplus from employment depends on their asset holdings. In other words, workers are in a better bargaining position if they hold more assets since they will be able to sustain a higher level of consumption during unemployment. See Krusell et al. (2010) for a more extensive discussion of this mechanism.

Firm owners also have access to the bond market where they can trade bonds with workers and with each other. I assume that all firm owners have the same endowments in period $t = 0$. Then, since the labor market is the only source of idiosyncratic risk and firm owners do not participate in the labor market, they are identical at all times. The representative firm owner chooses a sequence of consumption, bond holdings, investment, and capital $\{c_t^F, a_{t+1}^F, i_t^F, k_{t+1}^F\}_{t=0}^{\infty}$ to maximize expected lifetime utility given an initial endowment of bonds and shares (a_0^F, x_0^F)

$$\begin{aligned}
&\max_{\{c_t^F, a_{t+1}^F, i_t^F, k_{t+1}^F\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^F) \\
&\text{s.t. } c_t^F \leq (1 + r_t)a_t^F + \Pi_t^F - a_{t+1}^F + r_t^k k_t^F - T_t^F - \frac{\phi}{2}\left(\frac{i_t^F}{k_t^F} - \delta_k\right)^2 k_t^F - i_t^F, \\
&k_{t+1}^F = (1 - \delta_k)k_t^F + i_t^F, \\
&a_{t+1}^F \geq 0, \quad k_{t+1}^F \geq 0.
\end{aligned}$$

No-trade equilibrium Since the gross supply of the bond is zero and households cannot borrow, it must hold in equilibrium that households do not save, $a_t = 0$. This requires that the interest rate is low enough.

Proposition 9. *Consider the extended model described above. In equilibrium, $a_t = 0$ and it holds for the equilibrium interest rate*

$$\begin{aligned}
1 + r_{t+1} \leq & \frac{1}{\beta}\Bigg[\pi_t^{e|e} \frac{(1 - \tau_t)w_t}{(1 - \tau_{t+1})w_{t+1}} \\
& \times \left(1 + (1 - \gamma)(1 - \psi)\frac{\frac{w_{t+1}^N}{b_{t+1}} - 1 - (J_{t+1}(e) - J_{t+1}(u))}{1 + (1 - \psi)(J_{t+1}(e) - J_{t+1}(u))}\right) \\
& + \pi_t^{u|e} \frac{(1 - \tau_t)w_t}{b_{t+1}}\Bigg]^{-1}
\end{aligned} \tag{45}$$

Proof. Consider an employed worker in period t. The choice $a_{t+1} = 0$ is optimal only if

$$\begin{aligned}
\frac{1}{(1 - \tau_t)w_t} \geq & \beta\Bigg[\pi_t^{e|e} \frac{1}{(1 - \tau_{t+1})w_{t+1}} \left(1 + r_{t+1} + \frac{\partial w_{t+1}(a_{t+1})}{\partial a_{t+1}}(1 + \tau_{t+1})\right) \\
& + \pi_t^{u|e} \frac{1 + r_{t+1}}{b_{t+1}}\Bigg].
\end{aligned} \tag{46}$$

The derivative of the wage with respect to asset holdings, $\frac{\partial w_{t+1}}{\partial a_{t+1}}$, shows up because savings raise wages as they improve workers' bargaining position (Krusell et al. 2010). I now characterize this effect of savings on the wage. The Nash bargained wage is

$$w_t^N = \arg\max_w \psi \log \left(J_t^e(a,w) - J_t^u(a)\right) + (1-\psi)\log\left(J_t^F(w)\right),$$

where

$$\begin{aligned} J_t^e(a,w) = &\max_{a',\ell} \log(w(1-\tau_t) + (1+r_t)a - a') \\ &+ \beta\left[\pi_t^{e|e}(\ell)J_{t+1}^e(a',w') + \pi_t^{u|e}(\ell)J_{t+1}^u(a')\right] \end{aligned}$$

and

$$\begin{aligned} J_t^u(a) = &\max_{a',\ell} \log(b_t + (1+r_t)a - a') \\ &+ \beta\left[\pi_t^{e|u}(\ell)J_{t+1}^e(a',w') + \pi_t^{u|u}(\ell)J_{t+1}^u(a')\right] \end{aligned}$$

are the worker value functions of the extended model with savings.

The following first order condition implicitly defines the Nash wage

$$\begin{aligned} 0 = F(w_t^N, a) \equiv &(1-\psi)\left(J_t^e(a,w_t^N) - J_t^u(a)\right)\frac{w_t^N(1-\tau) + (1+r_t)a - a'}{1-\tau_t} \\ &- \psi\left(J_t^F(w_t^N)\right). \end{aligned}$$

Its derivates with respect to the Nash-wage and asset are

$$\begin{aligned} \frac{\partial F(w_t^N,a)}{\partial w_t^N} = &(1-\psi)\left[\left(\frac{\partial J_t^e(a,w_t^N)}{\partial w_t^N} - \frac{\partial J_t^u(a)}{\partial w_t^N}\right)\right. \\ &\times \frac{w_t^N(1-\tau) + (1+r_t)a - a'}{1-\tau} \\ &\left. + J_t^e(a,w_t^N) - J_t^u(a)\right] - \psi\frac{\partial J_t^F(w_t^N)}{\partial w_t^N} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial F(w_t^N,a)}{\partial a} = &(1-\psi)\left[\left(\frac{\partial J^e(w_t^N,a)}{\partial a} - \frac{\partial J^u(a)}{\partial a}\right)\right. \\ &\times \frac{w_t^N(1-\tau) + (1+r_t)a - a'}{1-\tau} \\ &\left. + (J_t^e(w_t^N,a) - J_t^u(a))\frac{1+r_t}{1-\tau}\right]. \end{aligned}$$

Using the implicit function theorem, it follows that

$$\frac{\partial w_t}{\partial a} = -(1-\gamma)\frac{(1-\psi)(1+r)}{1-\tau}\frac{1-(1-\tau_t)\frac{w_t^N}{b_t} + (J_t^e(w_t^N, a) - J_t^u(a))}{1+(1-\psi)(J_t^e(w_t^N, a) - J_t^u(a))}.$$

Substituting this into (46) with $a = 0$ and $w_t^N = w_t$ yields

$$\begin{aligned}\frac{1}{(1-\tau_t)w_t} \geq & \beta\left(1+r_{t+1}\right)\left[\pi_t^{e|e}(\ell_t)\frac{1}{(1-\tau_{t+1})w_{t+1}}\right.\\ & \times\left(1+(1-\gamma)(1-\psi)\frac{\frac{w_{t+1}^N}{b_{t+1}} - 1 - (J_{t+1}(e) - J_{t+1}(u))}{1+(1-\psi)(J_{t+1}(e) - J_{t+1}(u))}\right)\\ & \left. + \pi_t^{u|e}(\ell_t)\frac{1}{b_{t+1}}\right].\end{aligned}$$

Solving for the interest rate gives,

$$\begin{aligned}1+r_{t+1} \leq & \frac{1}{\beta}\left[\pi_t^{e|e}\frac{(1-\tau_t)w_t}{(1-\tau_{t+1})w_{t+1}}\right.\\ & \times\left(1+(1-\gamma)(1-\psi)\frac{\frac{w_{t+1}^N}{b_{t+1}} - 1 - (J_{t+1}(e) - J_{t+1}(u))}{1+(1-\psi)(J_{t+1}(e) - J_{t+1}(u))}\right)\\ & \left. + \pi_t^{u|e}\frac{(1-\tau_t)w_t}{b_{t+1}}\right]^{-1}.\end{aligned}$$

Here, ℓ_t is the effort choice of unemployed workers in the equilibrium of the main text where saving is ruled out. For this interest rate, the necessary condition for optimality of $a_{t+1} = 0$ is satisfied. □

For the calibration of the model, I assume that equation (45) holds with equality, i.e, the equilibrium interest rate is

$$\begin{aligned}1+r_{t+1} = & \frac{1}{\beta}\left[\pi_t^{e|e}\frac{(1-\tau_t)w_t}{(1-\tau_{t+1})w_{t+1}}\right.\\ & \times\left(1+(1-\gamma)(1-\psi)\frac{\frac{w_{t+1}^N}{b_{t+1}} - 1 - (J_{t+1}(e) - J_{t+1}(u))}{1+(1-\psi)(J_{t+1}(e) - J_{t+1}(u))}\right)\\ & \left. + \pi_t^{u|e}\frac{(1-\tau_t)w_t}{b_{t+1}}\right]^{-1}.\end{aligned} \tag{47}$$

This choice can be justified as the equilibrium interest rate in the limit as the supply of bonds goes to zero, which Werning (2015) labels the case of vanishing liquidity.

Condition (45) is only a necessary condition. It may not be sufficient for two reasons. First, it ensures that employed workers do not save, but unemployed workers might still do so if the job finding probability is high relative to the separation rate. In this case, unemployed workers have a stronger incentive to save than employed workers. We can obtain a condition similar to (45), that is necessary to rule out saving of unemployed workers. Second, because of the endogenous effort choice, households' expected utility is not necessarily concave in effort and assets. Starting from zero savings, a simultaneous increase in savings and decrease in effort may raise expected lifetime utility. Hence, I numerically verify that $a_t = 0$ is indeed an optimal choice for all households if the interest rate is given by (47).

B.4 News shock

The preceding discussion has highlighted the role of expectation about future productivity for the employment effect of public investment. The importance of expected future productivity can also be seen when comparing the employment effect of public investment to the change in employment that would result from a permanent change in productivity, defined as follows.

Definition 4 (Employment effect of (future) productivity)**.** *Denote by $N_t(\mathcal{Y}_0, z_0, z_1, \ldots)$ employment in period t in an equilibrium with initial conditions $\mathcal{Y}_0 = (N_0, w_0, K_0)$ and productivity sequence $\mathcal{Z} = (z_t)_{t=0}^{\infty}$. Consider a permanent increase in productivity in period T. The employment effect in t is defined as*

$$M_t^z(T, \mathcal{Y}_0, \mathcal{Z}) = \frac{\partial N_t(\mathcal{Y}_0, \ldots, z_{T-1}, xz_T, xz_{T+1}, \ldots)}{\partial x}|_{x=1}.$$

I get the following result

Proposition 10. *If the economy is in its steady state initially, then*

$$M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = \frac{\vartheta}{1 - \beta(1 - \delta_G)(1 - \rho)} \frac{\delta_G}{I^G} M_t^z(T, \mathcal{Y}_0, \mathcal{Z}).$$

The employment effect of public investment is proportional to the employment change in response to a permanent change in future productivity where the factor of proportionality depends on the elasticity of productivity with respect to public investment. For private agents, the announcement of the public investment expansion constitutes a news shock about productivity and, up to a constant factor, induces the same employment response.

Proof. Consider the productivity sequence $(z_k)_{k=0}^{\infty}$ with $z_k = z$ for $k < t$ and $z = xz$ for

$k \geq T$. The wage in period s is

$$w_s = \begin{cases} \gamma^s w_0 + (1-\gamma)\omega a^\alpha z^{\frac{1}{1-\alpha}} \frac{\gamma^s - 1}{\gamma - 1}, \text{ if } s < T \\ \gamma^s w_0 + (1-\gamma)\omega a^\alpha z^{\frac{1}{1-\alpha}} \left(\frac{\gamma^s - \gamma^{s-T+1}}{\gamma - 1} + x^{\frac{1}{1-\alpha}} \frac{\gamma^{s-T+1} - 1}{\gamma - 1}\right), \text{ if } s \geq T \end{cases}$$

and for $k < T$

$$\begin{aligned} \pi_k^{e|u} =& \zeta^{\frac{1}{\eta}} (1-\rho)^{\frac{1-\eta}{\eta}} \left(z^{\frac{1}{1-\alpha}} \kappa a^\alpha\right)^{\frac{\eta-1}{\eta}} \\ &\times \left[\sum_{s=k}^{T-1} (\beta(1-\rho))^{s-k} (1-\alpha) a^\alpha z^{\frac{1}{1-\alpha}} \right. \\ &+ \sum_{s=T}^{\infty} x^{\frac{1}{1-\alpha}} (\beta(1-\rho))^{s-k} (1-\alpha) a^\alpha z^{\frac{1}{1-\alpha}} \\ &- \sum_{s=k}^{\infty} (\beta(1-\rho))^{s-k} \gamma^s w_0 + \sum_{s=k}^{T-1} (\beta(1-\rho))^{s-k} \omega a^\alpha z^{\frac{1}{1-\alpha}} (\gamma^s - 1) \\ &+ \sum_{s=T}^{\infty} (\beta(1-\rho))^{s-k} \omega a^\alpha z^{\frac{1}{1-\alpha}} (\gamma^s - \gamma^{s-T+1}) \\ &\left. + \sum_{s=T}^{\infty} (\beta(1-\rho))^{s-k} \omega a^\alpha z^{\frac{1}{1-\alpha}} x^{\frac{1}{1-\alpha}} (\gamma^{s-T+1} - 1) \right]^{\frac{1-\eta}{\eta}} \end{aligned}$$

which can be simplified to

$$\begin{aligned} \pi_k^{e|u} =& \zeta^{\frac{1}{\eta}} (1-\rho)^{\frac{1-\eta}{\eta}} \left(z^{\frac{1}{1-\alpha}} \kappa a^\alpha\right)^{\frac{\eta-1}{\eta}} \\ &\left\{ \frac{(1-\alpha-\omega) a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} \left[1 + (\beta(1-\rho))^{T-k} (x^{\frac{1}{1-\alpha}} - 1)\right] \right. \\ &+ \gamma \frac{\omega a^\alpha z^{\frac{1}{1-\alpha}}}{1-\gamma\beta(1-\rho)} (\beta(1-\rho))^{T-k} (x^{\frac{1}{1-\alpha}} - 1) \\ &\left. - \frac{\gamma^k}{1-\gamma\beta(1-\rho)} (w_0 - \omega a^\alpha z^{\frac{1}{1-\alpha}}) \right\}^{\frac{1-\eta}{\eta}} \end{aligned}$$

I have that for $k < T$

$$\begin{aligned}\frac{\partial \pi_k^{e|u}}{\partial x}|_{x=1} =& \pi_k^{e|u} \frac{1-\eta}{\eta} \frac{1}{1-\alpha} \left\{ \frac{(1-\alpha-\omega) a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} \right. \\ & \left. - \frac{\gamma^k}{1-\gamma\beta(1-\rho)} (w_0 - \omega a^\alpha z^{\frac{1}{1-\alpha}}) \right\}^{-1} (\beta(1-\rho))^{T-k} \\ & \left(\frac{(1-\alpha-\omega) a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} + \gamma \frac{\omega a^\alpha z^{\frac{1}{1-\alpha}}}{1-\gamma\beta(1-\rho)} \right),\end{aligned}$$

If the wage in period 0 is at its steady state value $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$, I have for $k < T$

$$\begin{aligned}\frac{\partial \pi_k^{e|u}}{\partial x}|_{x=1} =& (\beta(1-\rho))^{T-k} \pi_k^{e|u} \frac{1-\eta}{\eta} \frac{1}{1-\alpha} \\ & \times \left(1 + \frac{\omega\gamma}{1-\omega-\alpha} \frac{1-\beta(1-\rho)}{1-\gamma\beta(1-\rho)}\right) > 0.\end{aligned}$$

Note that $1-\omega-\alpha > 0$ if $\pi_k^{e|u} > 0$. The short-run employment effect is

$$\begin{aligned}M_t^z(T, \mathcal{Y}_0, \mathcal{Z}) =& (1-N_0) \frac{\partial \pi_0^{e|u}}{\partial x} = (1-N_0)(\beta(1-\rho))^T \pi_0^{e|u} \frac{1-\eta}{\eta} \frac{1}{1-\alpha} \\ & \times \left(1 + \frac{\omega\gamma}{1-\gamma\beta(1-\rho)} \frac{1-\beta(1-\rho)}{1-\omega-\alpha}\right)\end{aligned}$$

If the economy is at the steady state initially, then the employment effect is

$$\begin{aligned}M_t^z(T, \mathcal{Y}_0, \mathcal{Z}) &= \sum_{k=0}^{t-1} (1-\rho-\pi^{e|u})^{t-k-1} (1-N) \frac{\partial \pi_k^{e|u}}{\partial x} \\ &= (\beta(1-\rho))^{T+1-t} (1-N) \pi^{e|u} \frac{1}{1-\alpha} \frac{1-\eta}{\eta} \\ & \times \frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^t}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))} \\ & \times \left(1 + \frac{\omega\gamma}{1-\gamma\beta(1-\rho)} \frac{1-\beta(1-\rho)}{1-\omega-\alpha}\right).\end{aligned}$$

The result then follows from the formula for the employment multiplier of public investment (22). □

C Additional Quantitative Results

Table 7: Business cycle moments with cross-correlations.

	U3	U5	Y	Inv.	Cons.	Wages	Lab. prod.	TFP
Standard dev.	0.121 (0.128)	0.121 (0.101)	0.018 (0.015)	0.112 (0.064)	0.019 (0.010)	0.005 (0.010)	0.010 (0.012)	0.012 (0.012)
Autocorr.	0.868 (0.884)	0.868 (0.943)	0.861 (0.845)	0.146 (0.820)	0.878 (0.784)	0.957 (0.746)	0.771 (0.758)	0.789 (0.798)
Corr. with ...								
U3	1.000 (1.000)	1.000 (0.992)	-0.946 (-0.843)	-0.614 (-0.725)	-0.961 (-0.667)	-0.453 (-0.360)	-0.819 (-0.223)	-0.879 (-0.485)
U5	1.000 (0.992)	1.000 (1.000)	-0.946 (-0.864)	-0.614 (-0.811)	-0.961 (-0.841)	-0.453 (-0.299)	-0.819 (0.106)	-0.879 (-0.315)
Y	—	—	1.000 (1.000)	0.707 (0.878)	0.993 (0.671)	0.543 (0.553)	0.943 (0.625)	0.970 (0.785)
inv.	—	—	—	1.000 (1.000)	0.662 (0.579)	0.134 (0.455)	0.714 (0.594)	0.766 (0.749)
cons.	—	—	—	—	1.000 (1.000)	0.589 (0.287)	0.915 (0.259)	0.939 (0.382)
wages	—	—	—	—	—	1.000 (1.000)	0.570 (0.564)	0.436 (0.613)
lab. prod.	–	—	—	—	—	—	1.000 (1.000)	0.978 (0.878)
TFP	—	—	—	—	—	—	—	1.000 (1.000)

Notes: Data moments are in parentheses. Variables are relative deviations from the HP trend with smoothing parameters 1,600. We use quarterly data from 1951q1 to 2019q4, except for moments involving U5, which use date from 1994q1 to 2019q4.

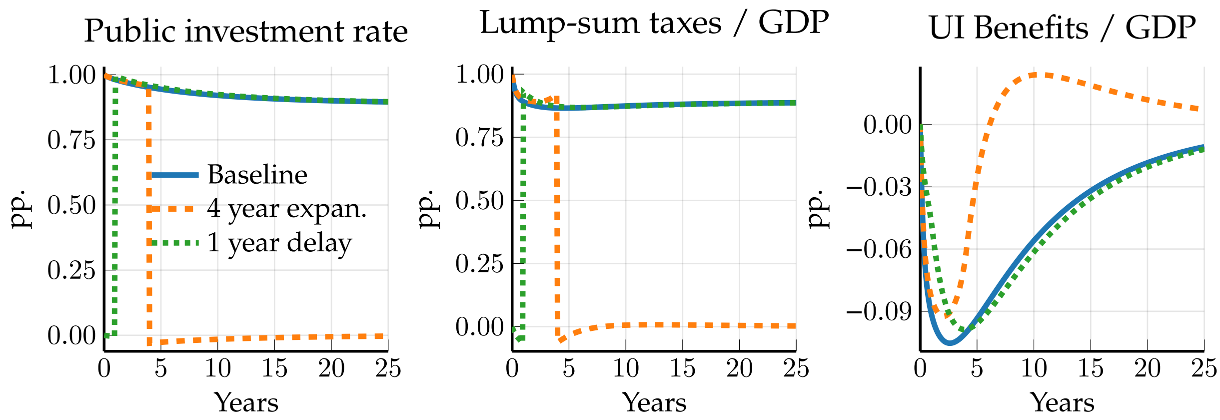

Figure 13: The fiscal response to the public investment expansion.

Table 8: Output multipliers of public investment at different horizons for different scenarios.

		1 year	2 years	3 years	Long run
Peak	baseline	0.47	0.77	1.03	2.92
	4 year expansion	0.43	0.70	0.91	2.92
	1 year delay	–	0.50	0.79	2.92
	recession	0.51	0.80	1.05	2.92
	labor tax financed	0.26	0.52	0.75	2.92
Cumulative	baseline	0.27	0.45	0.61	2.92
	4 year expansion	0.25	0.42	0.55	2.92
	1 year delay	–	0.42	0.54	2.92
	recession	0.31	0.49	0.64	2.92
	labor tax financed	0.13	0.27	0.40	2.92

Notes: Peak multiplier: maximum change in output over change in public investment over the respective horizon H, $\max_{h\leq H} \frac{\Delta Y_h}{\Delta I_h^G}$. Cumulative multiplier: cumulative change in output over horizon H over cumulative change in public investment over the same horizon, $\frac{\sum_{h\leq H} \Delta Y_h}{\sum_{h\leq H} \Delta I_h^G}$.

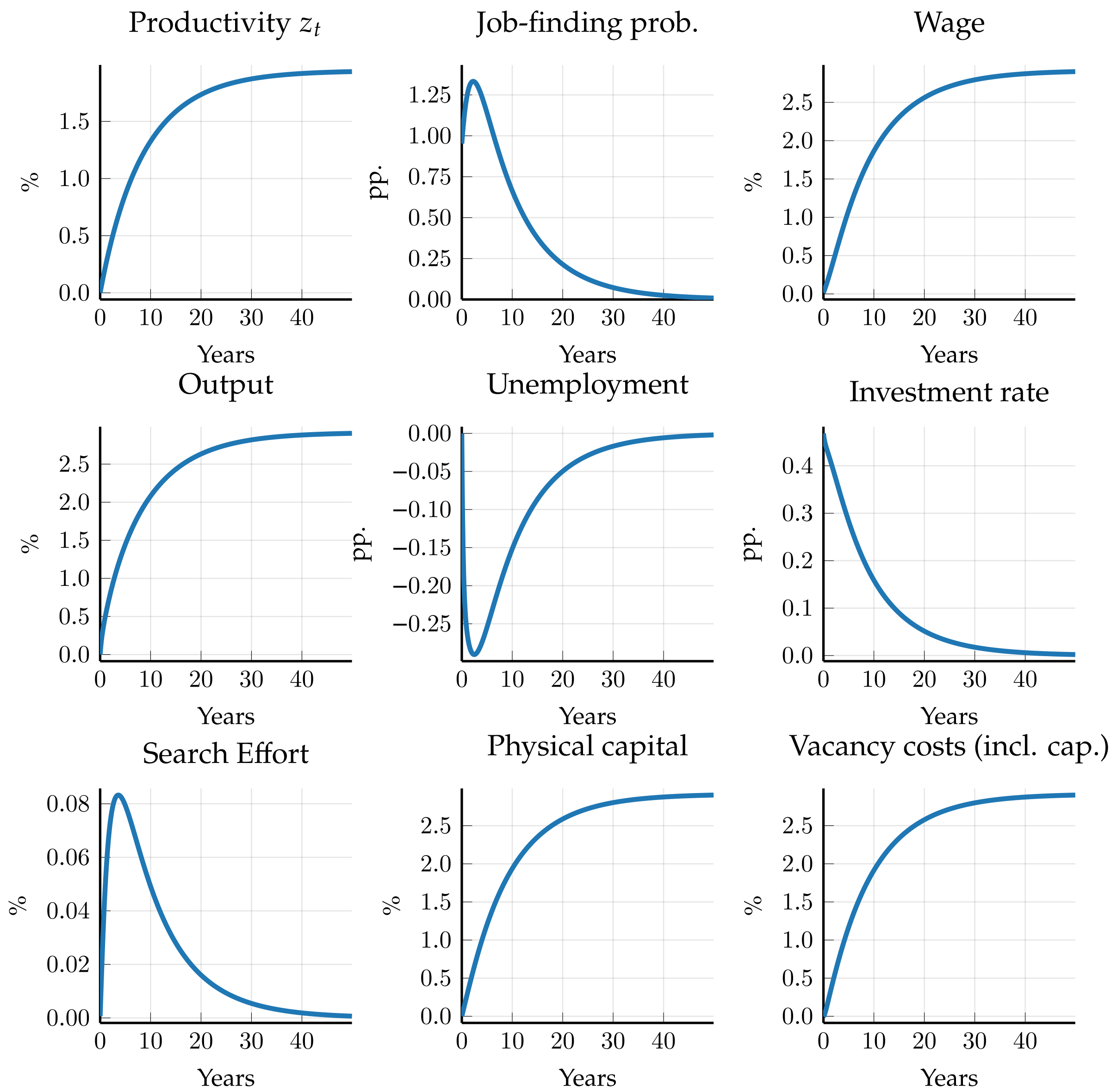

Figure 14: Long-run responses to a government investment program.

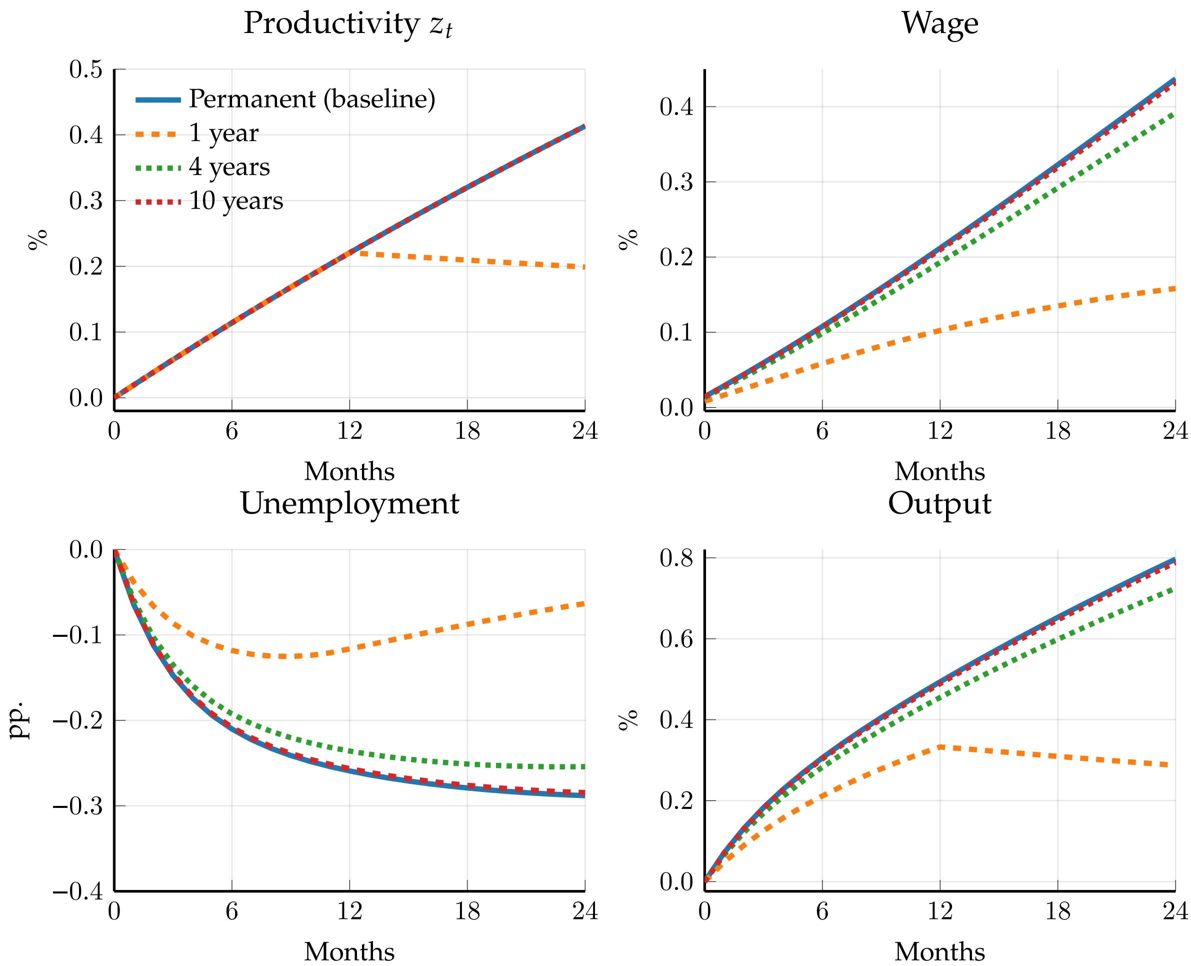

Figure 15: Responses to transitory expansion in public investment.

C.1 State dependence

For the case without an expansion in public investment, Figure 16 shows the evolution of unemployment, labor market tightness and wages starting from the two different initial conditions (boom and recession) described in the main text.

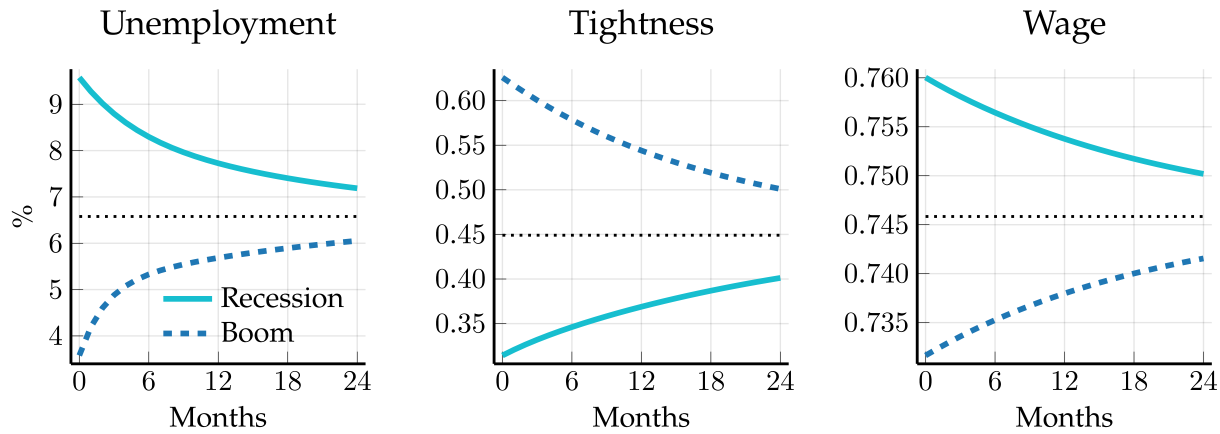

Figure 16: Unemployment, labor market tightness and wages in recession and boom.
Notes: The black dotted line denotes the steady state. Unemployment in percent, wages in units of the consumption good.

In the main text, I study the state dependence of the employment effect of public investment considering a recession that results from a joint positive shock to the separation rate and the wage level (and vice-versa for a boom). Here, I alternatively consider a recession due to a negative shock to productivity of one standard deviation,

$$\log A_0 = -0.0056$$

and accordingly, for a boom

$$\log A_0 = +0.0056,$$

after which productivity A_t evolves according to (29).

Figure 17 show the response of TFP, unemployment, labor market tightness and wages.

Qualitatively, I obtain the same result as in the main text—the employment effect of public investment is larger in a recession.

C.2 Alternative parameterizations

For the baseline calibration I have chosen the bargaining power of workers ψ such that the labor share is 64% as in the data. Alternatively, I could require that the bargaining power is such that vacancy creation is efficient in the steady state, i.e., the wage is given by (43). Note that the right term in (43) is zero in the steady state given our calibration strategy. The employment and wage response for a re-calibration of the model that requires workers

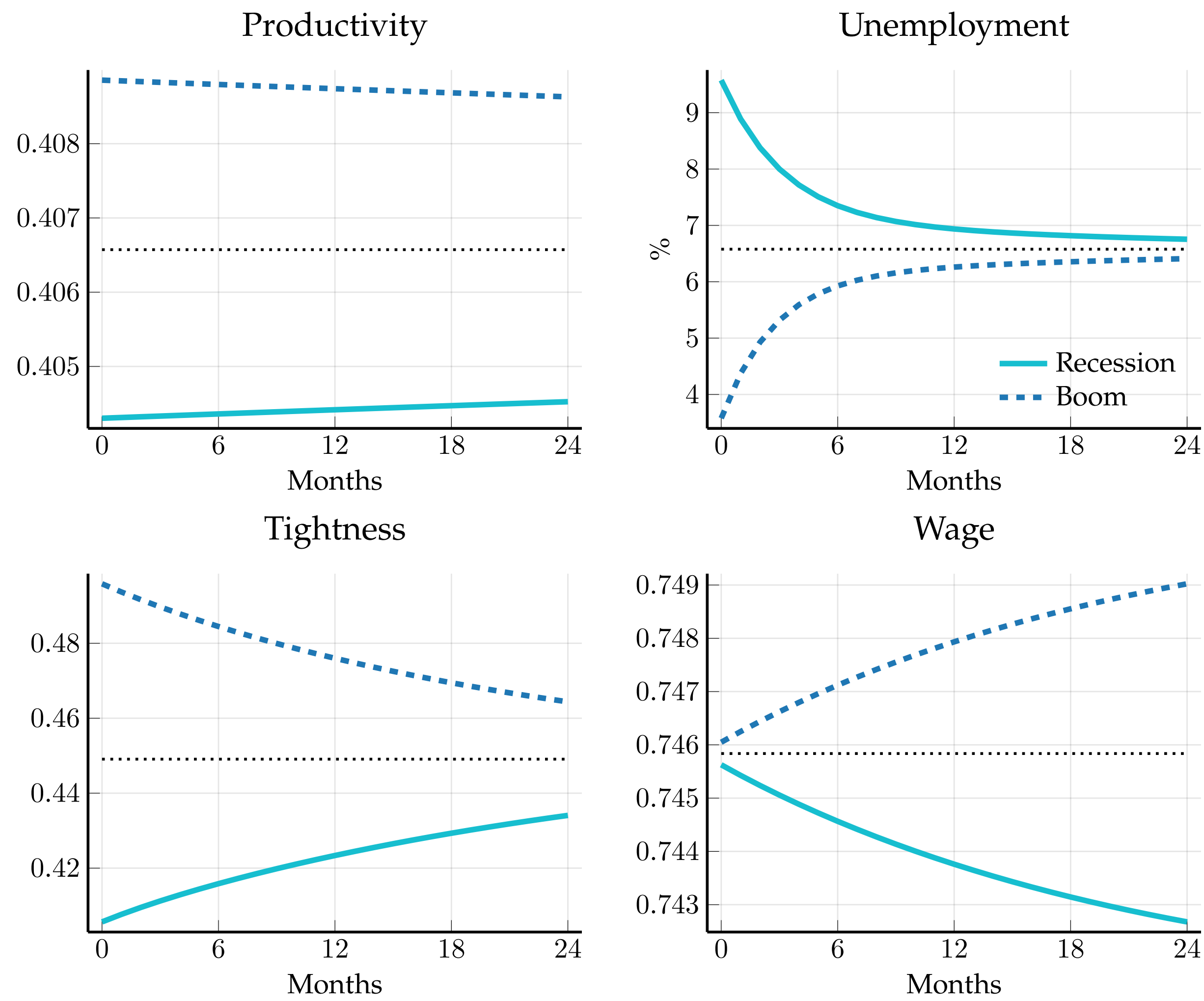

Figure 17: Responses to productivity shocks.

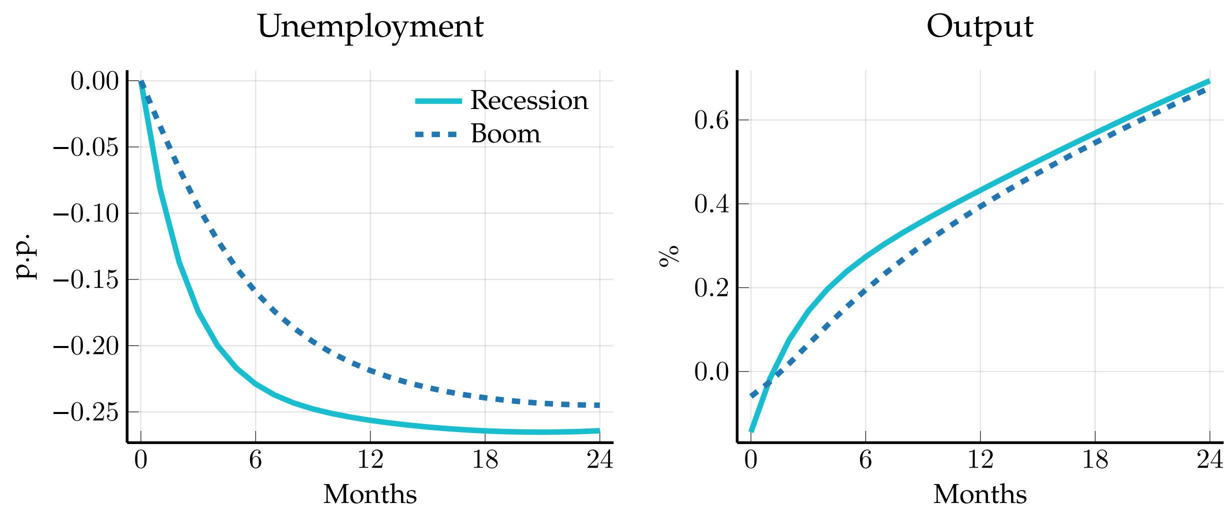

Figure 18: Responses of unemployment and output to permanent expansion in public investment in recession and boom.
Notes: Shown are the deviations from the paths without an expansion in public investment (see Figure 17)

bargaining power to implement efficient vacancy creation in steady state is shown by the dashed red line in Figure 19.

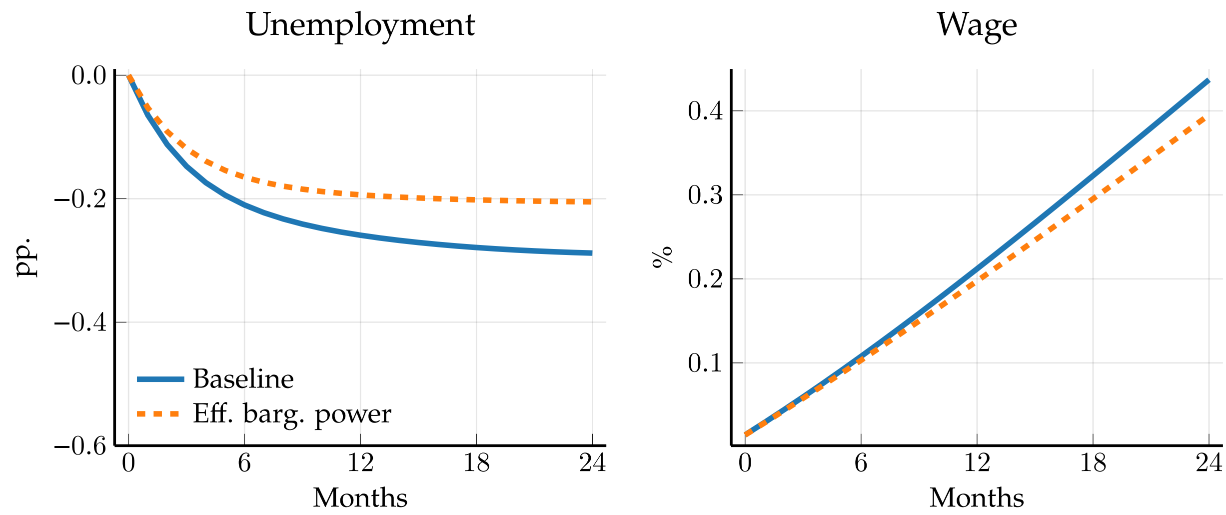

Figure 19: Response if bargaining power implements efficient vacancy creation in steady state.
Notes: Dashed red line: response of unemployment for calibration where workers' bargaining power is chosen to implement efficient level of vacancy creation in steady state.

Our baseline specification assumes that posting costs are proportional to labor productivity. The dotted red line in Figure 20 shows the short-run response of unemployment and wages if posting costs are constant instead. The dashed orange line shows the responses when capital adjustment costs are zero. The dashed green line shows the responses when capital adjustment costs are infinite, i.e., the private capital stock is constant.

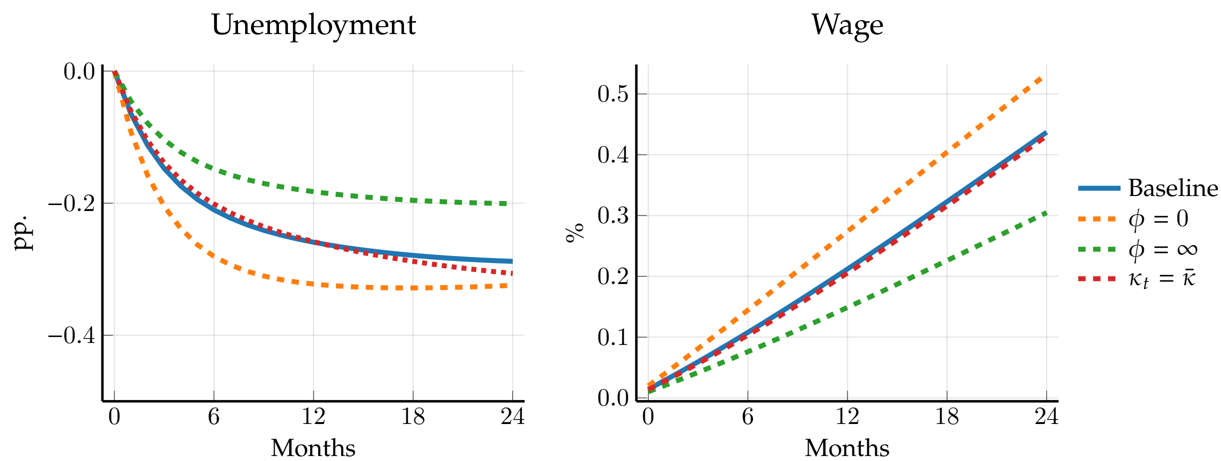

Figure 20: Responses without capital adjustment costs and with constant vacancy posting costs.
Notes: Dashed orange line: no capital adjustment costs. Dashed green line: infinite capital adjustment costs. Dotted red line: constant posting costs.

Figure 21 varies the degree of wage stickiness.

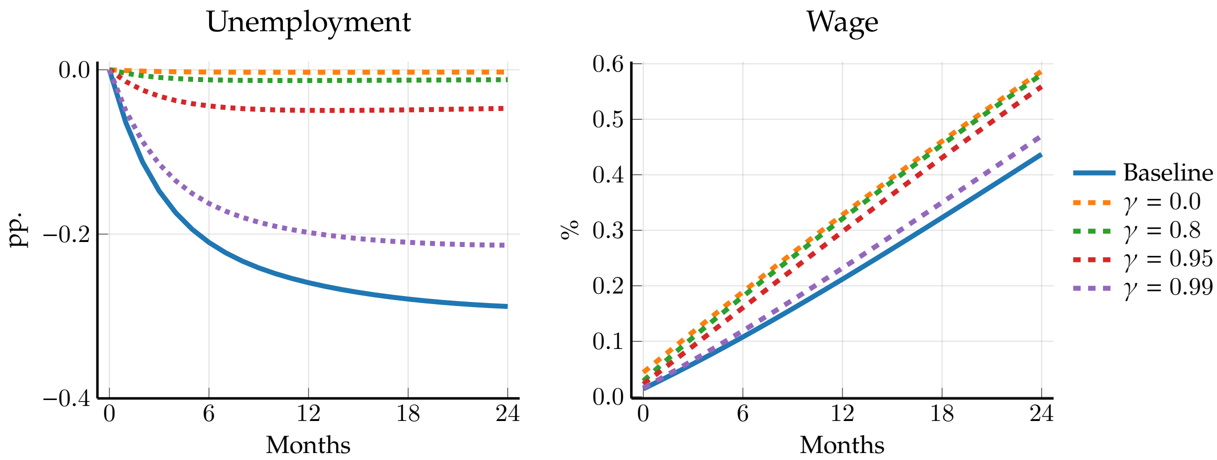

Figure 21: Responses for different degrees of wage stickiness.

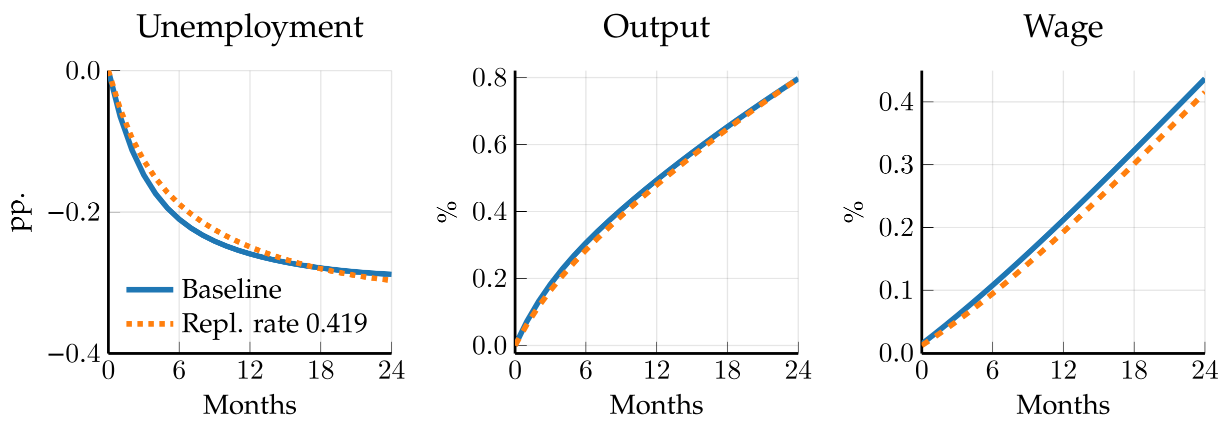

Figure 22: Responses with wage replacement rate of 41.9%.

Notes: Responses of unemployment, output and wages in a re-calibrated model with a (net) wage replacement rate of 41.9% Absent wage stickiness, this would imply a fundamental surplus of 60% as in Shimer (2005). The resulting calibration for the bargaining power of workers is 0.725, very close to the value of 0.72 in Shimer (2005).

C.3 Different household structure

A potential concern is that the quantitative results are driven by the household structure, in particular the separation of households into workers and firm owners. To alleviate this concern, I consider the following alternative household structure with one representative household. The representative household receives firms' profits, capital income, aggregate wages and unemployment benefits and invests in capital. This means, the representative household receives the same income as the sum of the incomes of the three household types in the benchmark model (employed workers, unemployed workers, firm owners). It further decides on the level of search effort of unemployed members of the household. Preferences are the same as workers' preferences in the benchmark model and the utility maximization problem is

$$\max_{C_t,\ell_t,I_t,K_{t+1},N_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{\ell_t^{1+\chi}}{1+\chi}(1-N_t) - d_{0,e}N_t \right)$$

subject to the budget constraint

$$C_t + I_t = (1-\tau_t)w_t N_t + b_t(1-N_t) + \Pi_t + r_t^k K_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta_k\right)^2 K_t - T_t^F$$

and the laws of motion for employment

$$N_{t+1} = (1-\rho)N_t + \pi_t^{e|u}(\ell_t)(1-N_t)$$

and capital

$$K_{t+1} = (1-\delta)K_t + I_t.$$

Note that this structure has different implications for the choice of effort than the benchmark model. Specifically, a higher level of aggregate consumption now has a negative effect on effort, whereas in the benchmark model, only the difference between lifetime utility of unemployed and employed workers determined the level of effort.

I further assume, that wages are set according to the same Nash bargaining protocol as before, i.e. even though the member of the representative household is insured perfectly and also receives capital and profit income, the member bargains over the wage as if it were an employed worker in the benchmark model.

As before, firms discount the future using their owner's discount factor when making hiring decisions. In the benchmark model this was the firm owner's discount factor, which was independent of aggregate consumption. In the representative agent model, the representative household owns the firm, and its discount factor depends on aggregate consumption.

I follow the same calibration strategy as in the benchmark model. Figure 23 compares the responses of key variables to a permanent expansion in public investment in this representative agent model to the benchmark model considered in the paper. The response of unemployment in the representative agent model is somewhat smaller than in the benchmark model but overall similar. Since the representative household has a motive to smooth consumption, the consumption decline on impact is smaller. This is achieved by a smaller expansion in private capital investment. Search effort now depends on the representative household's level of consumption as well as on the wage and the job-finding probability. It turns out that effort actually increases more than in the benchmark model. The reason is that consumption falls because of higher taxes and more investment which incentivizes the representative household to search more.

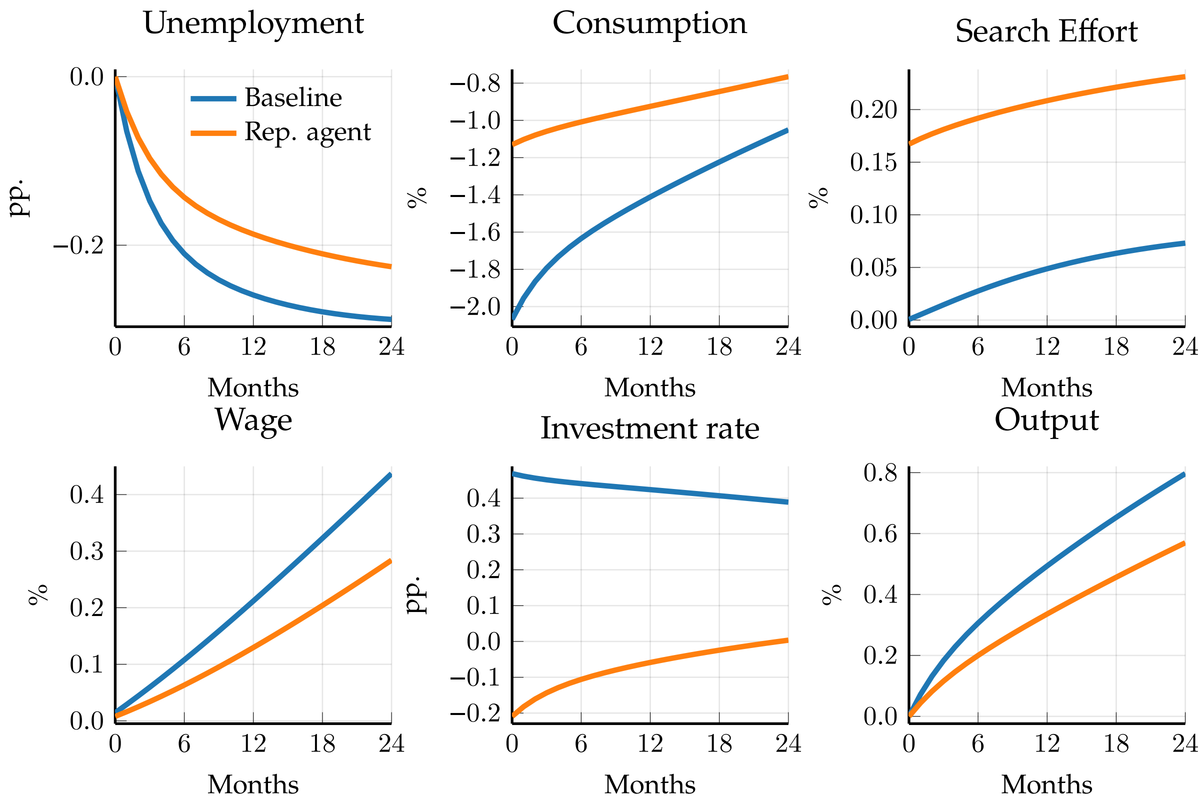

Figure 23: Responses to a permanent expansion in public investment in the benchmark model and the representative agent model.

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