The Short-Run Employment Effects of Public Infrastructure Investment

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Abstract

I study the stimulus effects of a permanent expansion of public investment that improves long-run productivity. Through an *anticipation effect on labor demand*, the policy change raises employment already in the short-run. In a model with search and matching labor market, I characterize the *employment multiplier of public investment* analytically and show that it is larger in a recession than a boom. Calibrated to the US, the model yields an increase in employment of 0.4 percentage points one year after a permanent expansion of public investment by 1% of GDP. The *anticipation effect* accounts for 65% of the employment gain.

Keywords: Public investment, infrastructure, labor markets, fiscal multiplier
JEL Codes: E22, E24, E62, H54

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1 Introduction

The "Infrastructure Investment and Jobs Act" appropriates $550 billion to additional federal infrastructure investment over the next five years. This would raise federal non-defense infrastructure investment from 0.7% of GDP in 2019 to about 1.3%, a level last seen in the 1970s. The EU also plans to expand infrastructure investment with the EU Recovery Fund allocating at least €383 billion to public investment supporting green and digital transformation. These expansions of public investment were proposed during a recession with high unemployment and a potential need for fiscal stimulus.

Many studies have found large positive effects of public investment on productivity and output *in the long run*, but it is a debated question whether a public investment program can also provide substantial short-term stimulus.[1] Summers (2009) argues in favor of expanding public investment during a recession. Ramey (2020) sees little expansionary effects, if any at all, and argues that government consumption likely has larger short-run effects than public investment.

This paper contributes to this debate. I take the positive long-run effects of public investment on productivity as given and show that they can lead to a substantial increase in employment in the short run. Quantitatively, a permanent expansion of public investment by 1% of GDP raises employment by 0.4 percentage points within one year.

The employment effects in the short run are due to an *anticipation effect on labor demand*. When public investment increases, firms anticipate higher private factor productivity and tighter labor markets in the future. They expand hiring immediately while it is still cheap since labor market tightness is low and workers can be found quickly. To the best of my knowledge, this paper is the first to analyze and quantify the anticipation effect of public investment on labor demand and its business cycle dependence.[2]

1. Aschauer (1989), Bom and Ligthart (2014), Bouakez et al. (2017), Cubas (2020), Munnell (1990), and Pereira and Frutos (1999) provide evidence on the long-run effects of public investment. I discuss this literature in detail in Section 4.

2. Merz (2010) mentions that positive productivity effects of government spending may lead to larger multipliers in a matching model but to date there is no theoretical

Labor market frictions are crucial for the increase in labor demand in response to higher future productivity. I model them following Mortensen and Pissarides (1994) and Pissarides (1985). A firm enters the labor market by posting a vacancy. After the vacancy has been filled, the firm produces output until worker and firm are separated. Importantly, worker-firm matches can last multiple periods. This renders firms' hiring decision forward-looking, allows firms to hoard labor, and gives rise to the anticipation effect on labor demand. In contrast, when firms hire labor period by period, as in much of the literature on public investment (e.g. Baxter and King 1993; Boehm 2020), there is no anticipation effect on labor demand.

In this paper, I focus on the anticipation effect of public investment on employment from rising long-run productivity. To isolate this effect, I abstract from two other channels through which public investment can affect employment. First, public investment may stimulate labor demand directly as the public sector hire workers to implement infrastructure projects (see Michaillat 2014). Second, public investment may affect employment through its effect on aggregate goods demand (see Rendahl 2016). Both of these channels are also present for unproductive spending, whereas the anticipation effect emphasized here is specific to public investment. Hence, the employment gain I find can be interpreted as the additional effect of government investment beyond what can be achieved through consumptive government spending.

In the first part of this paper, I analyze the employment effect of public investment theoretically. I assume that labor supply is constant to focus on the anticipation effect on labor demand. I define the *employment multiplier of public investment*: the change in employment following the announcement of a permanent expansion in public investment by one dollar starting at some point in the future.

The employment multiplier is strictly positive in the short run, although it may be zero in the long run. Thus, the positive employment multiplier is a transitional phenomenon: The increase in public investment and not its higher level causes the short-run increase in em-

analysis and quantification of this effect.

ployment. The employment multiplier differs between recessions and normal times, it is larger when unemployment is high. In this case, every additional vacancy only leads to a small increase in labor market tightness such that the congestion externality is small.

For the case where the economy is in the steady state initially, I derive an analytic expression for the employment multiplier. The formula highlights three main results: First, the multiplier is larger if public investment has greater long-run effects on productivity. As a result, the employment multiplier of public investment exceeds the multiplier of government consumption. Second, implementation delays reduce the employment multiplier. When they are longer, productivity only increases in the more distant future. Therefore, a worker hired today is less likely to be employed at the firm when the productivity effect of investment materializes. Hence, firms expand vacancy creation less strongly in the short run. Third, wage stickiness amplifies the employment effect; when wages rise more slowly following the increase in future productivity, labor hoarding is cheaper and firms expand vacancy creation in the short run more strongly.

These theoretical results rely on the assumption that labor supply is constant, which allows me to focus on the new mechanism in my model, the anticipation effect of public investment on *labor demand*. However, public investment could also affect *labor supply* in the short run. For example, Leeper et al. (2010) find that an increase in future productivity lowers labor supply through a wealth effect such that public investment has smaller output effects than unproductive spending. In my search and matching model, labor supply corresponds to search effort of unemployed workers. I show that the sign of the employment multiplier of public investment is theoretically ambiguous when workers chose effort endogenously. In response to an expansion in public investment, workers might increase or decrease search effort in the short run. Better job-finding prospects in the medium-run lead to lower search effort in the present, whereas more vacancies today trigger higher search effort. How employment responds to an expansion in public investment is thus a quantitative question to which I turn in the second part of this paper.

I calibrate the model to US data and consider a permanent expansion of public infrastructure investment by 1% of GDP. First, I assume that the public investment program is implemented as soon as it is announced and that it is financed with lump-sum taxes. The permanent expansion of public investment boosts the long-run level of productivity by 3%, which is reached after about 25 years. After one year, the expansion of public investment has increased productivity by only 0.35%, but unemployment is already 0.4 percentage points lower than before. The anticipation effect accounts for up to 65% of the employment gain after one year.

Then, I consider implementation lags. They reduce the employment response upon announcement of the expansion in public investment, but the response remains large. When one year passes between the announcement and the implementation of the investment program, unemployment still declines by 0.25 percentage points within the first year after announcement of the program.

When the government levies distortionary labor taxes to finance the additional public investment, the employment effect is smaller but still positive. The reduction in unemployment one year after the beginning of the investment program is still close to 0.25 percentage points.

Wage inertia is quantitatively important for the large employment gains in the short run. Under my calibration, wages increase almost in proportion to productivity. If wage inertia is smaller, expectations about higher future productivity lead to a stronger wage increase in the short run as workers demand higher wages. This makes labor hoarding more costly for firms and the employment effect is smaller.

Finally, the employment effect is more than 40% larger in a recession, where unemployment and profit margins are lower than in a boom.

My results imply that recessions are good times to announce a policy change towards more public infrastructure investment. The policy change can stimulate employment in the short run even if there are substantial implementation delays, and the employment reduction and the associated output gains are particularly large in a recession.

Related Literature A large literature in macroeconomics studies fiscal multipliers. Two strands of this literature are related closely to this paper. The first is the literature on the short-run effects of public investment. Baxter and King (1993), Boehm (2020), Leeper et al. (2010), and Ramey (2020) use structural models to quantify the output and employment effects of public investment. These studies find smaller short-run effects of public investment than of government consumption because the long-run productivity gains associated with public investment push down labor supply in the short-run through a positive wealth effect. Except for Ramey (2020), these papers consider frictionless labor markets. Firms' labor choice is static and independent of future productivity. Changes in expected future productivity due to public investment only directly affect the labor supply decision of workers. The model in Ramey (2020) features labor market frictions in the form of sticky wages, but labor demand is still a static decision. In contrast, I study a model with search frictions in the labor market in which labor demand depends on future productivity. I find that the short-run employment effects of public investment are significantly larger than those of unproductive government spending.

Empirical evidence on the short-run effects of public investment is sparse and ambiguous. In a panel of OECD countries, Boehm (2020) finds a short-run output multiplier of public investment close to zero, smaller than the multiplier of government consumption. The literature survey by Ramey (2020) largely focuses on evidence from the American Recovery and Reinvestment Act (ARRA) and concludes that public investment has at best a small positive effect on output and employment. In contrast, a meta study by Gechert (2015) finds the average estimated output multipliers of public investment to be 1.4, significantly larger than for government consumption. Auerbach and Gorodnichenko (2012) find a short-run output multiplier of public investment of 2.39 for the US, about twice as large as for government consumption. They also document a larger effect of public investment during recessions than expansions. Similarly, Clemens et al. (2022) find short-run output multipliers of public investment larger than 2 for Germany.

My model can help reconcile these seemingly contradictory empirical findings. I find that public investment only has a significant effect on output and employment in the short run if it boosts expected long-run productivity. This might not have been the case for the ARRA, which aimed to stabilize aggregate demand and primarily funded "shovel-ready" projects. If these projects were already planned and were only expedited, their effect on expected productivity and therefore employment would have been small.

I share the emphasis on frictional labor markets with a second strand of the literature on fiscal multipliers. Mitman and Rabinovich (2015) focus on unemployment benefit extensions, Monacelli et al. (2010) and Rendahl (2016) investigate government consumption, and Michaillat and Saez (2018) and Michaillat (2014) study public sector employment. I add an analysis of a different type of government spending, public investment, which has not been studied in the context of a search and matching labor market.

This paper is also related to the literature on news-driven business cycles following Beaudry and Portier (2006) who find anticipated future TFP growth to be an important driver of business cycles (Beaudry and Portier 2007; Schmitt-Grohé and Uribe 2012). In my model, public investment alters expectations about private future productivity and as such constitutes a news shock causing an expectations-driven boom.[3] Thus, it is related to Den Haan and Kaltenbrunner (2009) who study a model with matching frictions and find that news about higher future productivity can generate a boom in investment, hours worked, consumption and output. My paper differs in the following ways. First, I analyze the employment effect theoretically. Second, I consider anticipated changes in productivity caused by public investment. Since public investment is costly, the government has to raise revenues to finance it, and I study the effects of public investment under different assumptions about its financing, lump-sum taxes and distortionary labor taxes. The short-run employment effects of public investment are positive even if financed with distortionary taxes. Third, I show that the employment effects are substantially larger in recessions.

3. The employment effect of public investment financed with lump-sum taxes is proportional to the effect of a permanent productivity increase (see online Appendix).

Outline The remainder of the paper is structured as follows. Section 2 presents the model. In Section 3, I define the employment multiplier of public investment and analyze it theoretically. I calibrate the model in Section 4 and quantify the employment and output effects of public investment in Section 5. Section 6 concludes.

2 Model

The model features random search and matching in the labor market following Diamond, Mortensen and Pissarides (DMP model). Firms use private and public capital (infrastructure) in production. They rent private capital from households whereas the government provides public capital, used in production by all firms simultaneously. Time $t = 0, \dots, \infty$ is discrete and runs forever.

2.1 Firms, labor market, and production

There is a large number of firms. Each firm can hire a worker to produce output, y_t, using private capital, k_t, and public capital, K_t^G, according to the production function

$$y_t = A_t \left(K_t^G\right)^{\vartheta} k_t^{\alpha}. \tag{1}$$

Here, A_t is exogenous productivity and ϑ is the output elasticity of public capital. The public capital stock K_t^G is used by all firms simultaneously and there is no congestion externality. Since firms cannot be excluded from using it, the government provides the public capital stock. Equivalently, we can write the production function as $y_t = z_t k_t^{\alpha}$, where

$$z_t = A_t \left(K_t^G\right)^{\vartheta} \tag{2}$$

is total productivity of private factors (TFP). It depends positively on the public capital stock and is taken as given by private firms.

To hire a worker, a firm posts a vacancy at per-period cost κ_t. These costs are the foregone production of the workers involved in hiring and therefore proportional to labor productivity y_t,

$$\kappa_t = \bar{\kappa} \cdot y_t,$$

where $\bar{\kappa}$ is the labor required to open a vacancy. With posting costs proportional to labor productivity, unemployment is constant in the long run even if productivity grows over time. In contrast, if posting costs were constant, productivity growth would lead to a sustained decline in unemployment as the costs of posting a vacancy would fall relative to output of a filled vacancy.

Let v denote the measure of open vacancies and let L^u denote aggregate search effort, which is individual search effort of unemployed workers aggregated over all unemployed workers. In equilibrium, all unemployed households will search with the same intensity, such that aggregate search effort is individual search effort, ℓ, times the measure of unemployed workers U, $L^u = U\ell$. Vacancies v and aggregate search effort L^u determine the number of firm-worker matches formed in a given period according to the Cobb-Douglas matching function

$$M(L^u, v) = \zeta \left(L^u\right)^{\eta} \left(v\right)^{1-\eta}, \text{ with } \eta \in (0,1). \tag{3}$$

More matches are formed if firms create more vacancies, there are more unemployed workers, or if unemployed workers search with greater intensity.

When $M(L^u_t, v_t)$ matches are formed in period t, there are $\frac{M(L^u_t, v_t)}{v_t}$ matches for every vacancy. Since matching is random, every firm with an open vacancy finds a worker with probability

$$q^v_t(\theta_t) = \frac{M(L^u_t, v_t)}{v_t} = \zeta \theta_t^{-\eta},$$

where $\theta_t \equiv \frac{v_t}{L^u_t}$ denotes labor market tightness.

When a firm has filled its vacancy, it rents private capital from households at rate r^k_t, produces output according to (1), and pays wage w_t to its worker.

Worker-firm matches are dissolved with probability $\rho < 1$ and continue with the complementary probability $1-\rho$. The value of a firm with a filled vacancy is

$$J_t^F = \max_k \; z_t k^\alpha - w_t - r_t^k k + \beta \Big\{ \rho V_{t+1} + (1-\rho) J_{t+1}^F \Big\}, \tag{4}$$

where V_{t+1} is the value of an open vacancy in the next period defined below and β is the firm's discount factor.[4]

The first-order condition for the optimal choice of capital in (4) is that the rental rate for capital equals the marginal product of capital,

$$r_t^k = \alpha z_t k_t^{\alpha-1}. \tag{5}$$

Using the first-order condition, the value of a filled vacancy can be written as

$$J_t^F = (1-\alpha) z_t k_t^\alpha - w_t + \beta \Big\{ \rho V_{t+1} + (1-\rho) J_{t+1}^F \Big\},$$

and the value of an open vacancy is

$$V_t = -\kappa_t + \beta \left\{ q_t^v J_{t+1}^F + (1-q_t^v) V_{t+1} \right\}. \tag{6}$$

It consists of the costs of opening a vacancy and the expected return: with probability q_t^v, the firm finds a worker and receives the value of a filled vacancy, J_{t+1}^F.

Because any firm can open a vacancy, its value must be zero in equilibrium. This implies that equilibrium labor market tightness solves the job-creation equation

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta \left\{ (1-\alpha) z_{t+1} k_{t+1}^\alpha - w_{t+1} + (1-\rho) \frac{\kappa_{t+1}}{q_{t+1}^v(\theta_{t+1})} \right\}. \tag{7}$$

The left-hand side of (7) is the expected cost to fill a vacancy. In equilibrium, it has to equal the value of a filled vacancy on the right-hand side which consists of discounted profits in the next period plus the expected continuation value of the match, taking into account that the

4. The firm's discount factor equals the discount factor of its owner, which is constant because firm owners have linear utility.

match survives with probability $1 - \rho$.

Equation (7) highlights the forward-looking nature of firms' vacancy posting decision and contains the main intuition for the labor demand effect of public investment. A persistent expansion of public investment leads to a gradual rise in the public capital stock which raises private productivity z. This leads to an increase in future labor market tightness, and raises the average time to fill a vacancy in the future. The expected costs of filling a vacancy in the future rise. As (7) shows, firms respond to higher expected vacancy filling costs in the future by expanding hiring in the present, pushing up labor market tightness. The dependence of firms' vacancy posting decision on future productivity hinges on the assumption that the separation rate is smaller than one, $\rho < 1$. Only then can firms substitute hiring over time.

Let N_t denote the measure of producing firms which equals employment as each firm employs exactly one worker. Since every firm uses k_t units of capital, the aggregate capital stock is $K_t = k_t N_t$ and aggregate output is

$$Y_t = z_t k_t^{\alpha} N_t = z_t K_t^{\alpha} N_t^{1-\alpha} = A_t \left(K_t^G \right)^{\vartheta} K_t^{\alpha} N_t^{1-\alpha}. \tag{8}$$

This is the same aggregate technology as in Baxter and King (1993).

2.2 Households

The household side of the model consists of a unit mass of workers and a mass μ of homogeneous firm owners as in Broer et al. (2019) and Ravn and Sterk (2020). Workers participate in the labor market and receive labor income when employed. Unemployed workers decide on search effort. Firm owners do not participate in the labor market, their income consists of firms' profits and capital returns.

Workers Workers differ regarding their labor market status s_t, they are employed $s_t = e$ or unemployed $s_t = u$. Workers find and lose jobs stochastically, and I denote the probability that a worker transitions from labor market state s to s' by $\pi^{s'|s}$.

Unemployed workers exert search effort $\ell_t \geq 0$ to raise the probability of finding a job. Besides effort, the job-finding probability depends on labor market conditions summarized by labor market tightness θ_t, which is determined endogenously as described above. Since the number of new matches per unit of aggregate search effort in period t is $\frac{M(L_t^u, v_t)}{L_t^u}$, an unemployed worker exerting search effort ℓ_t finds a job with probability

$$\pi_t^{e|u}(\theta_t, \ell_t) = \frac{M(L_t^u, v_t)}{L_t^u}\ell_t = q_t^v(\theta_t)\theta_t\ell_t.$$

We can think of $\frac{M(L_t^u, v_t)}{L_t^u}$ as the number of matches per application sent. Since every application results in a match with the same probability (random matching), a worker who sent out ℓ_t applications, finds a job with probability $\pi_t^{e|u}(\theta_t, \ell_t) = \frac{M(L_t^u, v_t)}{L_t^u}\ell_t$.[5]

Recall, that matches between workers and firms are separated with probability ρ. Hence, the probability of losing a job is $\pi^{u|e} = \rho$.

Unemployed workers receive unemployment benefits b_t, whereas employed workers earn the wage w_t which is taxed at rate τ_t. Hence, workers face income risk. When a worker loses the job, net income falls from $(1 - \tau_t)w_t$ to b_t.

Workers consume their income in every period, they do not own assets to insure against a job loss,

$$c_t(s_t) = \begin{cases} (1 - \tau_t)w_t, & \text{if } s_t = e \\ b_t, & \text{if } s_t = u. \end{cases} \tag{9}$$

This is a strong assumption, but a large fraction of US households actually live hand-to-mouth, especially among the unemployed (Kaplan et al. 2014). Since the consumption level during unemployment determines the search effort decision of unemployed workers, I calibrate the wage replacement rate of unemployment benefits to yield a realistic consumption drop upon job loss.

Workers value consumption and dislike effort according to the per-

5. The job-finding probability $\pi_t^{e|u}(\theta_t, \ell_t)$ and the vacancy filling probability $q_t^v(\theta_t)$ could exceed one. I assume and verify that they are smaller than one in equilibrium.

period utility function

$$u(c, \ell, s) = \log(c) - d(\ell, s).$$

Employed workers do not exert search effort. However, since disutility from effort depends on the employment state, the utility specification can capture a fixed disutility from working with $d(0, e) > 0$.[6]

Workers choose effort to maximize expected lifetime utility,

$$\begin{aligned} &\max_{\{\ell_t(s_t), c_t(s_t)\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\log(c_t(s_t)) - d(\ell_t(s_t), s_t) \right) \middle| s_0, \{\ell_t(s_t)\} \right] \\ &\qquad \text{s.t. (9), } \ell_t(s_t) \geq 0 \text{ and given } s_0. \end{aligned} \tag{10}$$

Here, the expectation is taken with respect to the labor market state s_t which depends on the initial state s_0 and past effort choices $\{\ell_t(s_t)\}$. Expected lifetime utility of a worker in labor market state s in period t can be expressed recursively as

$$\begin{aligned} J_t(s) = &\max_{\ell, c} \log(c) - d(\ell, s) + \beta \sum_{s' \in \{e,u\}} J_{t+1}(s') \pi_t^{s'|s}(\ell, \theta_t) \\ &\text{s.t. } c = (1 - \tau_t) w_t \mathbb{1}_{s=e} + b_t \mathbb{1}_{s=u}. \end{aligned} \tag{11}$$

The first-order condition for the optimal effort choice is

$$\frac{\partial d(\ell, u)}{\partial \ell} = \beta \left[J_{t+1}(e) - J_{t+1}(u) \right] \frac{\partial \pi_t^{e|u}(\theta_t, \ell)}{\partial \ell}. \tag{12}$$

The left-hand side is the utility cost of marginally increasing effort. The right-hand side is the gain in expected lifetime utility from increasing effort. More search effort raises the probability of finding a job and thereby expected future income. Since all unemployed workers are identical, equilibrium search effort is the same for all unemployed workers.

Aggregate employment N_t evolves according to

$$N_{t+1} = (1 - \rho) N_t + \pi_t^{e|u}(\theta_t, \ell_t) U_t. \tag{13}$$

6. This is equivalent to employed workers exerting a fixed amount of work effort.

Recall that the total mass of workers is one and every worker is either employed or unemployed. Thus, employment N_t and unemployment U_t sum to one and U_t is the unemployment rate of workers.

Firm owners There is a measure μ of identical firm owners. The representative firm owner receives aggregate profits Π_t and owns the aggregate stock of private capital K_t, which follows the law of motion

$$K_{t+1} = (1 - \delta_k)K_t + I_t. \tag{14}$$

Here, δ_k is the depreciation rate of physical capital and I_t denotes aggregate private investment. Firm owners rent out the capital stock to firms at the rental rate r_t^k.

I assume that firm owners face quadratic capital adjustment costs,

$$\Phi(I_t, K_t) = \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta_k\right)^2 K_t.$$

Adjustment costs are needed for quantitatively realistic fluctuations of investment over the business cycle, but they do not substantially affect the main results on employment. The budget constraint of the representative firm owner is

$$I_t + C_t^F = r_t^k K_t + \Pi_t - T_t^F - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta_k\right)^2 K_t, \tag{15}$$

where T_t^F denotes lump-sum taxes and c_t^F total consumption of firm owners.

Firm owners are risk neutral and maximize lifetime utility given by

$$U^F = \sum_{t=0}^{\infty} \beta^t u^F(C_t^F) = \sum_{t=0}^{\infty} \beta^t C_t^F$$

subject to the budget constraint (15) and the law of motion for capital (14). The resulting first-order condition for capital is

$$1 + \phi\left(\frac{K_{t+1}}{K_t} - 1\right) = \beta\left[1 + r_{t+1}^k - \delta_k + \frac{\phi}{2}\left(\left(\frac{K_{t+2}}{K_{t+1}}\right)^2 - 1\right)\right]. \tag{16}$$

Wage Determination Many wages are consistent with an equilibrium in the search and matching labor market described so far. I assume that the wage in period t is a linear combination of the wage in the previous period and a target wage w_t^*,

$$w_t = \gamma w_{t-1} + (1-\gamma) w_t^*. \tag{17}$$

The target wage is determined by Nash bargaining between workers and firms,

$$w_t^* = \arg\max_{w} \left(J_t(e,w) - J_t(u)\right)^{\psi} \left(J_t^F(w)\right)^{1-\psi}, \tag{18}$$

where ψ is the bargaining power of workers.[7]

For the theoretical analysis in Section 3, I do not consider Nash bargaining but instead make the simplifying assumption that the target wage is a fixed fraction ω of match output,

$$w_t^* = \omega z_t k_t^{\alpha}, \tag{19}$$

where ω is such that the wage lies in the bargaining set so that both workers and firms are wiling to sustain the match.

The parameter γ in (17) governs the strength of wage inertia. Wages are completely fixed if $\gamma = 1$ and the wage always equals the target wage if $\gamma = 0$. Sticky wages are a common assumption at least since put forward by Hall (2005) as an empirically plausible way to resolve the observation by Shimer (2005) that the standard DMP model with Nash bargaining cannot match the counter-cyclicality of unemployment in the data. Hall (2003, 2005) also provides a micro foundation for the specific functional form (17). Pissarides (2009) challenges wage stickiness as a solution to the Shimer puzzle documenting that only wages of new hires matter for the volatility of unemployment and that these exhibit little inertia. However, Gertler et al. (2020) show that wages in new matches are not as flexible as previously thought once composition effects are accounted for, supporting the assumption of sticky wage.

7. In the definition of the value functions (11) and (4) the dependence on w was subsumed in the aggregate state of the economy indicated by the time subscript t.

2.3 Government

The government pays unemployment benefits b_t and invests in public capital. Public investment I_t^G determines the public capital stock which follows the law of motion

$$K_{t+1}^G = (1 - \delta_G)K_t^G + I_t^G, \tag{20}$$

where δ_G is the depreciation rate of public capital. To finance its expenditures, the government collects lump-sum taxes on firm owners T_t^F and taxes labor income at rate τ_t.

The government's per-period budget constraint reads

$$I_t^G + U_t b_t = T_t^F + \tau_t w_t N_t, \tag{21}$$

where U_t is the number of unemployed workers and $N_t = 1 - U_t$ is the number of employed workers in period t. The left-hand side of the government's budget constraint are government expenditures for public investment and unemployment benefits. The right-hand side captures total tax revenues.

I formally define an equilibrium in the appendix.

3 Theoretical Analysis: Anticipation Effect on Labor Demand

In this section, I analyze the employment multiplier of public investment theoretically focusing on labor demand. I am interested in the change in employment in some period $t \geq 0$ that is brought about by a public investment program. The program is announced in period 0 and permanently raises public investment starting in period $T \geq 0$. Hence, T denotes the implementation lag of public investment. I define the employment multiplier of public investment as follows.

Definition 1 (Employment multiplier of public investment). *Denote by $N_t(\mathcal{X}_0, I_0^G, I_1^G, \dots)$ employment in period t in an equilibrium with initial conditions $\mathcal{X}_0 = (N_0, w_0, K_0^G, K_0)$ and public investment sequence $\mathcal{I}^G = \left(I_s^G\right)_{s=0}^{\infty}$.*

Consider a permanent expansion in public investment starting in period T. The employment multiplier of public investment in t is

$$M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = \frac{\partial N_T(\mathcal{X}_0, I_0^G, \dots, I_{T-1}^G, I_T^G + x, I_{T+1}^G + x, \dots)}{\partial x}|_{x=0}.$$

The employment multiplier tells us how much employment changes in period t when it is unexpectedly announced in period 0 that public investment will rise by 1 dollar in all periods after T. Figure 1 illustrates the employment multiplier of public investment graphically. The dots indicate the initial paths of public investment and employment, the crosses indicate the paths after the expansion of public investment. The difference between the two employment paths at a given point in time is the employment multiplier.

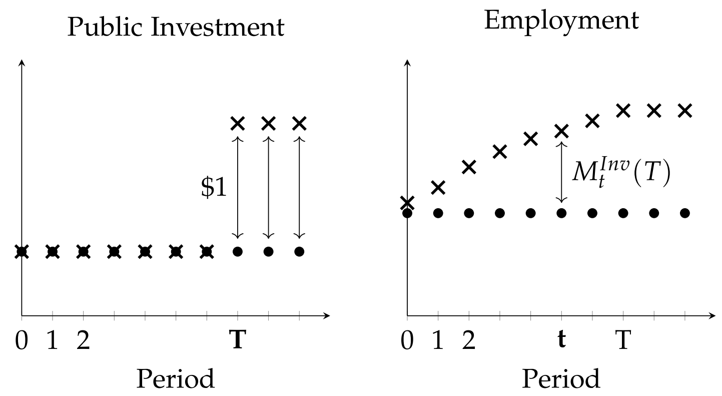


Figure 1: Graphical illustration of the employment multiplier of public investment $M_t^{Inv}(T)$.
Notes: Dots: initial paths; crosses: paths after policy change.

I make three assumptions.

Assumption 1. *Search effort is fixed at $\ell_t(u) = 1$.*

Hence, I focus on the role of labor demand for the employment multiplier of public investment. I consider elastic search effort in the quantitative analysis in Section 5 and find that search effort contributes little to the employment effect.

Assumption 2. *The target wage is a fixed fraction of output and taken as given by firms, i.e., the target wage is given by* (19).

Assumption 2 implies that there is no feedback from vacancy posting to wages through Nash bargaining which simplifies the analysis. Given the optimal capital choice, the job-creation equation (7), together with the law of motion for employment (13) and the accumulation equation of public capital (20), is then sufficient to characterize the employment multiplier.

Assumption 3. *Capital adjustment costs are zero, $\phi = 0$.*

This assumption simplifies the law of motion for capital as it eliminates the dependence of the optimal capital choice k_{t+1} on the current capital stock as well as on planned future capital. The choice for k_{t+1} then only depends on expected productivity in $t+1$ but not on past and future capital choices.

What happens to employment when the government announces a permanent expansion in public investment? Firms anticipate higher productivity in the future which increases the value of a filled vacancy but does not increase the cost of posting a vacancy. Therefore, firms post more vacancies and employment rises.

Importantly, the short-run employment effect is a dynamic phenomenon. Unless wages are completely rigid, public investment does not affect employment in the long run. The reason is that wages and posting costs grow in proportion to labor productivity in the long run. Hence, while output from a match is higher, so are all costs and the incentives to post vacancies are unchanged. Yet, employment still increases temporarily along the transition to the new steady state with high public investment. The reason is that hiring costs are fixed in the short-run whereas the return from a filled vacancy increases with future productivity. Proposition 1 formalizes these points.

Proposition 1 (Positive short-run employment multiplier of public investment)**.** *Suppose that $I_t^G = \delta_G K_0^G$ for all $I_t^G \in \mathcal{I}^G$ and that the initial wage is at least at the steady state level $w_0 \geq \omega \left(\frac{\alpha\beta}{1-\beta(1-\delta_k)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}}$. Then, under assumptions 1–3, the employment multiplier of public investment is*

(i) positive, $M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) > 0$,

(ii) zero in the long-run, $\lim_{t\to\infty} M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = 0$*, if wages are not completely rigid,* $\gamma < 1$.

Proof. See Appendix. □

The employment multiplier at the steady state can be characterized succinctly.

Proposition 2. *Suppose that the economy is in a steady state with* $I_t^G = \delta_G K^G$ *for all* $I_t^G \in \mathcal{I}^G$ *and assumptions 1–3 hold. Then, for* $t + 1 \leq T$*, the employment multiplier of public investment is*

$$\begin{aligned} M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) =& \vartheta \frac{(\beta(1-\rho))^{T+1-t}}{1-\beta(1-\delta_G)(1-\rho)} \frac{\delta_G}{I^G} \frac{1}{1-\alpha} \\ & \times \left[1 + \frac{\gamma\omega(1-\beta(1-\rho))}{(1-\beta\gamma(1-\rho))(1-\alpha-\omega)} \right] \\ & \times \frac{1-\eta}{\eta} U \pi^{e|u} \frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^t}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))} > 0 \end{aligned} \tag{22}$$

with $\pi^{e|u} = \zeta^{\frac{1}{\eta}} \left(\frac{\bar{\kappa}(1-\beta(1-\rho))}{\beta(1-\alpha-\omega)} \right)^{\frac{\eta-1}{\eta}}$.

Proof. See Appendix. □

Proposition 2 clarifies the mechanism through which public investment affects employment in the short run and how the size of the employment multiplier depends on the fundamentals of the economy. Let us consider the employment multiplier in the first period, for $t = 1$. In this case, equation (22) reads

$$\begin{aligned} M_1^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = & \overbrace{\frac{\vartheta\beta^T(1-\rho)^T}{1-(1-\delta_G)\beta(1-\rho)} \frac{\delta_G}{I^G}}^{\text{Semi-elasticity of match productivity w.r.t. public investment}} \overbrace{\frac{1}{1-\alpha}}^{\text{Elasticity of profits w.r.t. productivity}} \\ & \times \left[1 + \underbrace{\frac{\gamma\omega(1-\beta(1-\rho))}{(1-\gamma\beta(1-\rho))(1-\alpha-\omega)}}_{\text{Wage stickiness}} \right] \frac{1-\eta}{\eta} U \pi(e|u). \end{aligned} \tag{23}$$

The first factor is the semi-elasticity of the present value of expected match productivity with respect to public investment. This drives the employment effect—public investment raises productivity over the expected course of a worker-firm match, which leads to vacancy creation by forward-looking firms. The productivity effect and thereby the employment multiplier are larger if the output elasticity of public capital, ϑ, is higher.

This is not surprising. If the output elasticity of public capital is larger, the future marginal product of labor and therefore future labor demand, labor market tightness and search costs increase more in response to an expansion in public investment. As a result, firms expand hiring in the short run more strongly. However, a larger output elasticity of public capital can have the opposite effect in a standard RBC model, where labor demand is a static decision and public investment affects employment by shifting labor supply (see Ramey 2020). The reason is that public investment raises household wealth more strongly if the output elasticity of public capital is higher. In the short run, this leads to a reduction in labor supply, hours worked, and output compared to a case with a low output elasticity of public capital.

The term $\frac{1}{1-\alpha}$ is the elasticity of current-period profits with respect to current productivity under flexible wages. Together with the term labeled "Wage stickiness" in (23) it determines how match output translates into firm profits. The investment program has a stronger effect on employment if profits respond more strongly to changes in productivity, which is the case if α is larger and if wages are more rigid.

The wage stickiness term is zero if wages are fully flexible ($\gamma = 0$) but the employment multiplier is still strictly positive. Even if wages adjust to higher labor productivity immediately, an increase in expected future productivity raises the expected present value of match profits, but it leaves the costs of posting a vacancy unchanged. This makes it more profitable for firms to post vacancies. Even though the employment multiplier is positive even when wages are flexible, it is larger if wages are stickier. The reason is that for more rigid wages, an expected increase in future productivity does not translate into a proportional increase in wages immediately so that per-period profits from a filled vacancy are expected to increase temporarily.

Finally, the employment multiplier depends on how strongly employment responds to additional vacancy creation, which is determined by the last term in (23). It depends on the elasticity of the matching function with respect to vacancies, $1 - \eta$, and on initial unemployment U. If the matching elasticity is high, additional vacancies translate into relatively more matches and employment increases more strongly.

Discount factor and separation rate affect the employment multiplier through two channels. First, they enter the first term in (23), the elasticity of match output with respect to public investment. Second, the discount factor and separation rate matter for the employment multiplier because they determine the importance of wage stickiness.

Suppose first, that there is no wage stickiness, $\gamma = 0$. In this case, a higher discount factor and a lower separation rate unambiguously increase the employment effect as both facilitate labor hoarding. When the discount factor is higher, firms value the increase in productivity in the (distant) future relatively more compared to additional costs of hiring and hoarding labor that are incurred in the near future. Hence, the employment effect of public investment is larger. If the separation rate is low, it is more likely that workers hired today will remain with the firm in the future. This makes it easier to substitute hiring intertemporarily when future costs of filling a vacancy increase as a result of tighter labor markets. This leads to a larger employment effect.

When wages are sticky, there is an opposing channel through which the discount factor and the separation rate affect the employment multiplier. A higher discount factor as well as a lower separation rate reduce wage stickiness term in (23) leading to a smaller employment multiplier. When wages are sticky, they remain low initially after the expansion in public investment such that profits increase more strongly at first. However, wages adjust to higher levels of productivity and lower the profit margin over time. In the long run, profits are unaffected by wage stickiness. When the separation rate is low or the discount factor is high, profits in the distant future are relatively more important for the total match surplus. Hence, the fact that wages remain low initially is of little importance for firms' vacancy creation. This is why, higher discount factors and lower separation rates can dampen the response of vacancy creation to an expansion in public investment. When wages are completely rigid ($\gamma = 1$) the discount factor and separation rate cancel out in the wage stickiness term. In this case, wages never adjust to higher productivity and the relative importance of wage payments in the distant future does not affect the present value of expected profits and vacancy creation. For the intermediate case with some wage stickiness, the overall effect of discount factor and separation rate on the employment effect of public investment is theoretically ambiguous. Quantitative analyses suggest that the employment effect increases with the discount factor and declines with the separation rate.

The employment multiplier at a given point in time depends negatively on the implementation lag T. If the lag is long, firms expect productivity to increase only in the very distant future and the program has a small effect on employment in the near future. For a given steady state job finding probability $\pi^{e|u}$, the degree to which the implementation lag matters depends on the discount factor β and the separation rate ρ.

Proposition 2 also shows that the employment multiplier increases with t, the time since the investment program has become known. The reason is twofold: First, as t increases the increase in productivity comes closer which raises the value of a filled vacancy and leads to more hiring. Second, more time has passed since news about higher future productivity became known such that firms' expansion in hiring has had more time to reduce unemployment.

3.1 Business cycle dependence

Are the employment effects of public investment different if the government announces the expansion in public investment during a recession? To shed light on this question, I investigate how the employment multiplier depends on two defining features of recessions, high unemployment and temporarily weak labor demand.

Public investment induces an increase in labor market tightness and the individual job-finding probability of unemployed workers, $\pi_t^{e|u}(\theta_t)$.

As can be seen from the law of motion for employment,

$$N_{t+1} = (1-\rho)N_t + \pi_t^{e|u}(\theta_t)U_t,$$

if the number of unemployed workers is larger, a given increase in the job finding probability benefits more workers and aggregate employment increases more strongly.

Another intuition for the effect of unemployment on the employment multiplier comes from firms' vacancy creation. When unemployment is high, any additional vacancy has only a small effect on the vacancy filling probability of other firms. For example, suppose labor market tightness is one, i.e., there is one vacancy for every unemployed worker. If there is only one unemployed worker, an additional vacancy doubles labor market tightness. In contrast, with ten unemployed workers, an additional vacancy increases labor market tightness by only 10%. In the second case, the additional vacancy will have a much smaller effect on the expected costs of all other firms to fill a vacancy than in the first case; the congestion externality is small when unemployment is high. Hence, vacancy creation expands more in response to an increase in public investment, and the employment multiplier of public investment is larger.

A second feature of recessions that is important for the short-run employment effect of public investment is weak labor demand, i.e., low labor market tightness and a small job-finding probability of unemployed workers. In the model, labor demand is low if the wage is high relative to productivity which means profit margins are small. Thus, I study how the short-run employment effect of public investment depends on the wage relative to productivity. To develop the main intuition, consider the job-creation equation (7) in period 0. It can be written as

$$\frac{\kappa_0}{q^v(\theta_0)} = \beta(y_1 - w_1 + (1-\rho)J_2^F), \tag{24}$$

where y_1 is labor productivity in period 1 and w_1 is the wage in period 1. The variable J_2^F is the value of a filled vacancy in period 2. For now, I interpret period 2 as the long run and I suppose that public investment raises the value of a match in the long run $\mathrm{d}J_2^F > 0$. The job-creation equation yields

$$\mathrm{d}q^v(\theta_0) = -\frac{\beta(1-\rho)}{\kappa_0} q^v(\theta_0)^2 \mathrm{d}J_2^F = -\frac{(1-\rho)\kappa_0}{\beta(y_1 - w_1 + (1-\rho)J_2^F)^2}\mathrm{d}J_2^F < 0.$$

As can be seen from the first equality, the vacancy filling probability declines relatively more if labor market tightness is low and the vacancy filling probability, $q^v(\theta_0)^2$, is high, i.e., if labor demand is weak. This is the case if the wage, w_1, is high relative to labor productivity, y_1, such that the value of a match is relatively small. In this case, the same increase in the long-run value of a match leads to a relatively larger effect on the total value of a match and thereby on the vacancy filling probability.

The corresponding change of the job-finding probability in response to an increase in the long run value of a match is

$$\mathrm{d}\pi_0^{e|u}(\theta_0) = q^{v\prime}(\theta_0)\theta_0 \mathrm{d}\theta_0 + q^v(\theta_0)\mathrm{d}\theta_0 = \frac{\eta - 1}{\eta}\theta_0 \mathrm{d}q^v(\theta_0) > 0.$$

We know from above that the change in the job finding probability, $\mathrm{d}q^v(\theta_0)$, is larger when labor demand is weak. But labor market tightness is lower, too, which has a negative effect on the employment multiplier. This is because, for a given level of unemployment, the same relative increase in labor market tightness corresponds to relatively few additional vacancies when labor market tightness is low.

Overall, the effect of weaker labor demand on the job finding probability is theoretically ambiguous, it depends on the elasticity of the matching function with respect to vacancies.

The next proposition shows that these intuitions carry over to the full model.

Proposition 3 (Business cycle dependence of employment multiplier)**.** *Suppose that* $I_t^G = \delta_G K_0^G$ *for all* $I_t^G \in \mathcal{I}^G$ *and that the wage is at the steady state level* $w_0 = \omega\left(\frac{\alpha\beta}{1-\beta(1-\delta_k)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}}$*. If assumptions 1–3 hold, then, for* $t+1 \leq T$*, the employment multiplier of public investment is*

(i) increasing in initial unemployment, $\frac{\partial M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G)}{\partial U_0} > 0$

(ii) increasing in the initial wage $\frac{\partial M_t^{Inv}(T,\mathcal{X}_0,\mathcal{I}^G)}{\partial w_0} \geq 0$ *if* $\eta > 0.5$.

Proof. See Appendix. □

3.2 The Role of Search Effort

So far, I focused on labor demand assuming that workers' search effort is perfectly inelastic. I now relax this assumption and show that when search effort is elastic, the sign of the employment multiplier is theoretically ambiguous—employment may increase or decrease following an expansion of public investment. The reason is that workers might reduce search effort which lowers the job-finding probability. This labor supply effect can potentially outweigh the positive effect of public investment on labor demand, such that employment decreases.

Formally, it holds that the change in the job-finding probability is

$$\mathrm{d}\pi_t^{e|u}(\theta_t,\ell_t) = \underbrace{\frac{\partial \pi_t^{e|u}(\theta_t,\ell_t)}{\partial \theta_t}\mathrm{d}\theta_t}_{\text{Labor demand } (>0)} + \underbrace{\frac{\partial \pi_t^{e|u}(\theta_t,\ell_t)}{\partial \ell_t}\mathrm{d}\ell_t}_{\text{Labor supply } \left(\lesseqgtr 0\right)} . \tag{25}$$

The first term is the labor supply effect, emphasized thus far. The second term is the labor demand (search effort) effect. It can be decomposed as,

$$\frac{\partial \pi_t^{e|u}(\theta_t,\ell_t)}{\partial \ell_t}\mathrm{d}\ell_t = \left(\underbrace{\frac{\partial \pi_t(\theta_t,\ell_t)}{\partial \theta_t}\mathrm{d}\theta_t \frac{1}{\ell_t}}_{\substack{\text{Effect of current labor} \\ \text{demand } (>0)}} + \underbrace{\frac{\pi_t(\theta_t,\ell_t)}{\ell_t}\frac{\mathrm{d}\Delta_{t+1}^{eu}}{\Delta_{t+1}^{eu}}}_{\substack{\text{Effect of future labor} \\ \text{demand } (<0)}} \right) \frac{\frac{\partial d(\ell_t,u)}{\partial \ell_t}}{\frac{\partial^2 d(\ell_t,u)}{\partial \ell_t^2}}, \tag{26}$$

where $\Delta_{t+1}^{eu} \equiv J_{t+1}(e) - J_{t+1}(u)$. Two opposing forces drive the response of search effort, captured by the two summands in the parentheses. First, the expansion in public investment leads to an expansion in the number of vacancies (the labor demand effect), which increases the marginal benefit of search effort. Second, public investment raises future job-finding probabilities, which makes it less important to find a job now and discourages workers from searching. Theoretically, the latter effect can be large enough to outweigh the positive effect of public investment on labor demand and the associated crowding in of labor supply. In this case, the employment multiplier is negative. The next proposition establishes this result.

Proposition 4 (Employment Multiplier with Elastic Search Effort)**.** *Suppose assumption 1 does not hold, i.e., search effort is not perfectly inelastic. Then, the employment multiplier can be positive or negative.*

Proof. See Appendix. □

Hence, I calibrate the model to quantify the employment effect of public investment. It turns out, that the labor demand effect strongly dominates the labor supply effect and employment increases following an expansion of public investment.

4 Calibration

To quantify the employment effect, I calibrate the model to the US economy with a period length of one month. The calibration is targeted at the steady state of the model. However, two parameters are irrelevant for the steady state, the degree of wage stickiness, γ, and ϕ, which governs the capital adjustment costs. For these, I pick values previously used in the literature and validate this choice by comparing the business cycle moments generated by the model to those in the data.

Technology Regarding the production technology, I set $\alpha = 0.33$ and assume the monthly depreciation rate of physical capital is $\delta_k = 0.00874$, which corresponds to 10% annually. Following Baxter and King (1993), the depreciation rate of public capital is also set to $\delta_G = 0.00874$. For the output elasticity of public capital, ϑ, the meta study by Bom and Ligthart (2014) points to an elasticity of 0.12 in the long-run, Bouakez et al. (2017) find 0.065 and Cubas (2020) finds 0.09. I decide on an intermediate value of 0.1 which is also considered in Leeper et al. (2010) and Leduc and Wilson (2013).[8] This is a conservative choice, other empirical

8. In addition to $\vartheta = 0.1$, Leeper et al. (2010) also consider $\vartheta = 0.05$.

studies have found substantially larger values than those above. For example, Aschauer (1989) finds 0.39 and Pereira and Frutos (1999) report 0.63 as a general equilibrium elasticity which according to Ramey (2020) corresponds to a value for ϑ of 0.39. Finally, I choose A to normalize labor productivity to one, $(1-\alpha)zk^{\alpha} = 1$.

Government I set the public investment rate to 2.9%, the US average between 1990 and 2019, and the labor tax rate τ to 30%. I assume that unemployment benefits are proportional to the net wage, $b_t = \bar{b}(1-\tau_t)w_t$, and set the replacement rate $\bar{b}$ to 70%. This is higher than the average replacement rate in the US, usually found to be close to 40%. However, it implies a decline in consumption expenditures upon becoming unemployed close to the estimates of Chodorow-Reich and Karabarbounis (2016) from the Consumer Expenditure Survey, which lie between 28% for food, clothing, recreation and vacation and 21% for food. Lump-sum transfers to firm owners are then chosen to ensure a balanced government budget.

Labor market The calibration of the labor market parameters follows Shimer (2005) matching the transition probabilities between employment and unemployment.

I estimate these transition probabilities using CPS microdata from January 1994 to December 2020 following Shimer (2012).[9] I use the unemployment concept U-5 from the Bureau of Labor Statistics (BLS) which includes marginally attached workers. This definition resembles the one in the model most closely, as all workers who are not employed are considered unemployed irrespective of how intensely they search. Results are very similar when I calibrate the model using U-3 unemployment instead.

Table 5 gives an overview of the estimation results. I find a monthly separation probability of 1.9% which directly informs the choice of the separation parameter ρ. It implies that jobs last about 52 months on average. Hagedorn and Manovskii (2008) use a slightly higher but comparable number of 2.6% that leads to an average job duration of 38

9. See the online Appendix for a detailed description of the estimation approach.

months.

The parameter ρ is crucial for the size of the employment effect as it determines how long firms can expect a match to last (see Proposition 2). In the model, the rate at which matches are dissolved equals the rate at which workers become unemployed or leave the labor force, but this need not be the case if there are job-to-job transitions. However, Hyatt and Spletzer (2016) document that average tenure has risen since the 1980s and median job tenure of employed workers was around 4.5 years in 2012, even longer than the median tenure of about three years implied by my choice for ρ.

For the monthly job finding probability I estimate a value of 26.9%. In contrast to the separation probability, the job finding probability $\pi_t^{e|u}$ is determined endogenously in the model and I match the estimated value by choosing the remaining labor market parameters as follows.

I set the elasticity of the matching function with respect to unemployment to $\eta = 0.3$. This is on the lower end of the range of empirical estimates surveyed in Petrongolo and Pissarides (2001) but still larger than 0.245 chosen in Hall (2005).

I chose a value of $\psi = 0.4016$ for workers' bargaining weight in order to match a labor share of 64%.

Den Haan et al. (2000) find a vacancy filling probability of $q^v = 71\%$. According to the job-creation equation (7), this requires $\bar{\kappa} = 0.8187$. It remains to calibrate the matching efficiency ζ and the disutility from effort. Regarding the latter, I assume that $d(\ell, s) = d_1 \frac{\ell^{1+\chi}}{1+\chi} + d_{0,s}$. I set $d_{0,u} = 0$ as a normalization and choose $d_{0,e}$ such that in the steady state there is no difference between the disutility from working and searching. This means that search effort and other non-pecuniary costs of unemployment such as lower social status offset the utility gain from more leisure, an assumption also made in McKay and Reis (2021). The matching efficiency ζ and the disutility parameter d_1 are not separately identified which is why I normalize $d_1 = 1$. I then choose $\chi = 5.6073$ to obtain a micro elasticity of the job finding probability with respect to unemployment benefits of –0.5.[10] This elasticity is in line with direct

10. In online Appendix A.2, I derive χ in terms of the micro elasticity of the job finding probability with respect to unemployment benefits.

empirical evidence in Chetty (2008) who obtains an estimate of –0.53. It is also in the range from –0.6 to –0.2 considered in Landais et al. (2018). I set $\zeta = 0.5584$ to match my estimate for the monthly job finding probability of 26.9%.

Discount factor I calibrate the discount factor such that the assumed hand-to-mouth behavior of workers is optimal in an extended model where workers can save in a risk-free bond. In Appendix B.3, I describe the extended model and show that workers are hand-to-mouth if the interest rate on the bond is at most

$$1 + r_{t+1} = \frac{1}{\beta}\left(\left[\pi_t^{e|e}\frac{(1-\tau_t)w_t}{(1-\tau_{t+1})w_{t+1}}\varphi_{t+1} + \pi_t^{u|e}\frac{(1-\tau_t)w_t}{b_{t+1}}\right]\right)^{-1} \tag{27}$$

with

$$\varphi_t = 1 - (1-\gamma)(1-\psi)\frac{1 - \frac{w_t^N}{b_t} + J_t(e) - J_t(u)}{1 + (1-\psi)\left(J_t(e) - J_t(u)\right)}. \tag{28}$$

I set the monthly discount factor to $\beta = 0.9930$ to obtain an annual interest rate of 1% according to equation (27).[11] Note that with $\varphi_{t+1} = 1$ the right-hand side of (27) is the standard formula for the intertemporal marginal rate of substitution between consumption today and tomorrow. The term φ_{t+1} captures an additional savings motive which arises because asset holdings affect the bargaining position of workers. Inspection of (28) shows that this motive is absent if wages are completely rigid ($\gamma = 1$) or if workers have the entire bargaining weight so that they receive the total surplus regardless of their asset holdings ($\psi = 1$).[12] Due to the precautionary savings motive and the effect of savings on the bargaining position, the discount factor is lower than under complete markets which leads to a relatively smaller employment effect (see the discussion in the previous section).

Table 1 provides an overview of the calibrated parameters. In the steady state, the unemployment rate is 6.58%. For comparison, the av-

11. Note that since workers face unemployment risk and firm owners do not, workers always have a higher willingness to save for a given discount factor.

12. See Krusell et al. (2010) for a detailed investigation of this effect on savings and the labor market.

Table 1: Baseline calibration.

Parameter		Value	Description	Target or source
Technology	ϑ	0.10	output elas. public capital	see text
	α	0.33	output elas. private capital	standard
	δ_G	0.0087	public capital depreciation	10.0% p.a.
	δ_k	0.0087	private capital depreciation	10.0% p.a.
	A	0.3576	productivity	normalization
	ϕ	15	capital adjustment costs	see text
Labor market	η	0.3	matching elasticity	PP01
	ψ	0.4016	worker bargaining weight	labor share 64.0%
	ρ	0.0189	separation probability	1.9% (own estimate)
	ζ	0.5584	matching efficiency	vacancy fill prob. 0.71
	$\bar{\kappa}$	0.8187	posting costs (labor)	job-finding prob. 0.269
	γ	0.9930	wage stickiness	see text
Preferences	β	0.9930	discount factor	interest rate 1.0% p.a.
	χ	5.6073	search elasticity	$\mathrm{d}\log \pi^{e\|u}/\mathrm{d}\log b = -0.5$
	$d_{0,e}$	0.0492	disutility from working	$d(\ell(u), u) = d(0, e)$
Government	τ	0.3	labor tax rate	standard
	$\bar{b}$	0.7	wage replacement rate	see text
	$\frac{I^G}{Y}$	0.029	public investment rate	average 1990–2019

Notes: PP01 stands for Petrongolo and Pissarides (2001).

erage U-5 unemployment rate from 1994 to 2020 was 6.86%. The (private) physical investment rate is 18.7%, close to the average of 17.3% observed in the data since 1990.

Finally, I set the parameter γ, which governs the extent of wage stickiness, to 0.993. This choice is also considered in Shimer (2010) who argues that it leads to a reasonable volatility of unemployment over the business cycle. I pick $\phi = 15$ for the capital adjustment cost parameter. As shown in the next subsection, for these choices, the model is able to replicate the volatility of unemployment and investment observed in the data for a realistic process of productivity. In the online Appendix, I investigate the role of wage stickiness γ and capital adjustment costs ϕ for the size of the employment multiplier (Figures 19 and 18).

4.1 Business cycle properties

I want to compare the standard deviation and quarterly autocorrelation of unemployment, output, investment, and labor productivity generated by the model to the data. In the data, I compute these moments for the relative deviations from a long-run trend obtained using an HP filter with smoothing parameter 1,600. I use data from the first quarter of 1951 to the fourth quarter of 2019. All moments shown in the first two rows of Table 2 are close to those found in the literature. In particular, the estimates of the standard deviation and autocorrelation of the U-3 unemployment rate are very close to those in Hagedorn and Manovskii (2008) who report 0.125 and 0.870, respectively. Since the calibration focuses on the broader measure of U-5 unemployment, I also report the respective moments for this variable. Relative to U-3 unemployment, it exhibits a slightly lower standard deviation of 0.101 and a higher autocorrelation of 0.943. Standard deviation and autocorrelation of labor productivity are also very close to the estimates in Hagedorn and Manovskii (2008) who find 0.013 and 0.765.

In order to assess the model's ability to replicate these moments, I assume that the public capital stock is constant and that A_t follows an AR(1) process in logs

$$\log A_t = \rho \log A_{t-1} + \nu_t, \quad \text{with } \nu_t \sim N(0, \sigma_\nu^2). \tag{29}$$

For the baseline calibration, I set $\rho = 0.9870$ and $\sigma_\nu = 0.0054$ such that standard deviation and autocorrelation of quarterly TFP in the model match the data. Here, I fix unemployment benefits at the steady state level. In reality, benefits depend on the individual labor market history. Thus, benefits grow with wages in the long run which is why I assume that benefits are proportional to wages in the next section, when I investigate the employment effects of a permanent expansion in public investment. Here, I only consider short-run fluctuations, so that a constant level of benefits is a good approximation to observed benefit schemes. Results are very similar when I assume that benefits are proportional to wages.

The last two rows of Table 2 show that the volatility of unemployment and output in the model are close to the data, although the volatility of unemployment is still slightly lower than observed in the data. As pointed out by Shimer (2005), it is difficult for the DMP model to match

Table 2: Overview of business cycle moments

		U-5	U-3	Y	Inv	Wages	Lab. prod.	z
Data	Std. dev.	0.101	0.128	0.015	0.065	0.010	0.012	0.012
	Autocorr.	0.943	0.886	0.845	0.821	0.744	0.761	0.797
Model	Std. dev.	0.081	–	0.017	0.090	0.008	0.011	0.012
	Autocorr.	0.848	–	0.846	0.248	0.947	0.789	0.791

Notes: All model moments are for relative deviations from the HP trend of the series aggregated to quarterly frequency. I use quarterly data from 1951:I to 2019:IV.

the volatility of unemployment. My model is able to generate a volatility similar to the data mainly because of the relatively high degree of wage inertia. Nevertheless, the volatility of wages in the model is only slightly lower than in the data. Despite capital adjustment costs, the volatility of private investment is still larger in the model than in the data, but the order of magnitude is the same. Table 6 in the online Appendix shows that the model also matches the cross-correlations between the variables reasonably well.

5 Quantitative Analysis

I assume first that the economy is in its steady state initially. Then, the government announces a permanent expansion in government investment by 1% of GDP in period zero, financed with lump-sum taxes on firm owners. In the long run, the program increases the public capital stock and thereby raises private factor productivity (z_t) by 3%.

Figure 2 shows the responses of key variables to the announcement of the government investment program over the first two years.[13] The solid blue lines depict the baseline scenario in which public investment increases at the same time the program is announced such that private productivity starts to rise in the first period.

Productivity of private factors increases almost linearly over the first two years of the program after which it is 0.63 percent higher than before. The increase in productivity brought about by public invest-

13. See Figures 12 and 11 in the online Appendix for the long-run responses and the corresponding fiscal policy.

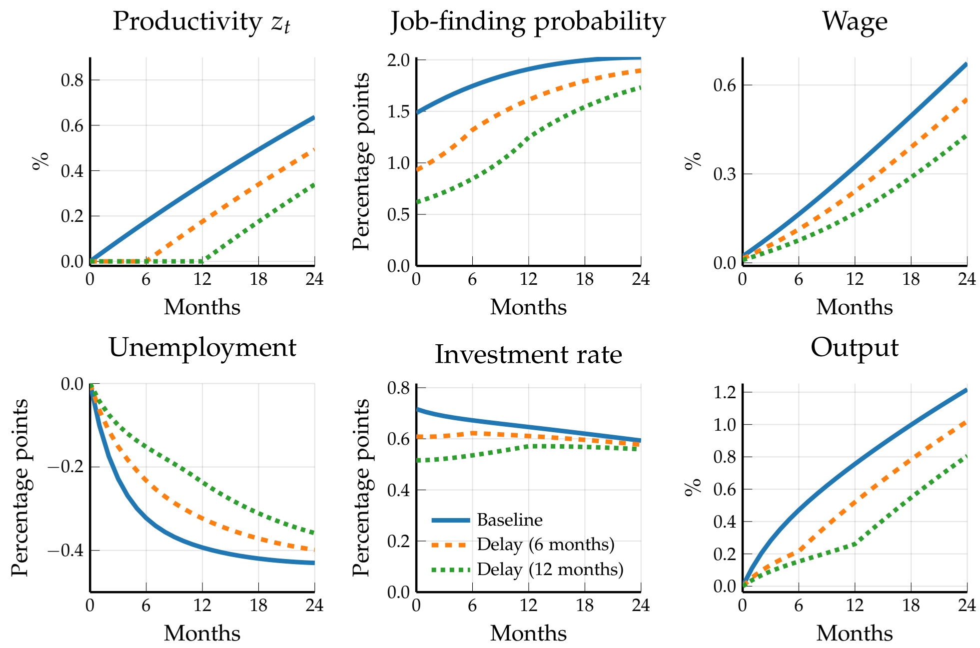


Figure 2: Short-run responses to public investment program.
Notes: Responses to a permanent expansion of public investment by 1% of initial GDP for different implementation delays as deviations from the initial steady state.

ment has a substantial effect on unemployment and output. With the start of the program, firms expand vacancy creation such that the job-finding probability increases by 1.5 percentage points on impact. The increase in the job-finding probability lowers the unemployment rate by 0.4 percentage points after twelve months. The private investment rate increases by about 0.7 percentage points on impact and then quickly returns to a permanently elevated level 0.6 percentage points above the one without the investment program. As a consequence of higher productivity, increased hiring, and private investment, output is about 0.8% higher after one year.

Wages also increase substantially and are 0.32% percent higher after one year than without additional public investment. This might be surprising at first given the seemingly high degree of wage inertia with $\gamma = 0.993$. The reason that wages still respond strongly is that the higher job-finding probability improves the bargaining position of workers such that the Nash bargaining wage increases substantially (see Figure 5 and the discussion below). During the first years after the start

Table 3: Employment multipliers after one year for different scenarios, jobs created per $ millions public investment.

Baseline	Recession	Delay 6 months	Delay 12 months	Labor tax financed
2.49	3.06	2.02	1.43	1.11

of the program, the Nash wage exceeds the new long-run wage, which leads to a much faster increase of the wage than would be obtained by substituting the new long-run Nash wage into the wage rule and iterating forward.[14]

The first column of Table 3 shows the corresponding employment multiplier of public investment as defined in Section 3. It amounts to 2.5 additional jobs created per million dollars of yearly public investment. The multiplier may appear small compared to recent empirical estimates of the multiplier of overall government spending, for example in Chodorow-Reich et al. (2012), Wilson (2012), and Serrato and Wingender (2016) who find employment multipliers of government spending between 8 and 38 jobs per one million in spending. This comparison is misleading for two reasons. First, the empirical estimates do not account for the quality of the job such that the additional jobs might be primarily low-paying jobs. For example, the estimated local income multiplier in Serrato and Wingender (2016) is 1.7 to 2, which is much less than the one that would be obtained if every created job paid the average wage. In contrast, jobs are homogeneous in my model such that the newly created jobs pay the average wage.

Second, the papers cited above estimate "local" multipliers which may be very different from aggregate multipliers (Ramey 2011). "Local" multipliers do not capture the general equilibrium effects associated with a nationwide expansion in public spending, which could dampen the employment effects. Aggregate-level estimates of government spending on employment that account for general equilibrium effects are rare. Monacelli et al. (2010) find that additional spending of 1% of GDP lowers unemployment by 0.43 percentage points after one year,

14. Compare the discussion in Hall (2003, Section V.C).

Table 4: Output multipliers of public investment.

	1 year	2 years	3 years	Long run
Peak	0.71	1.18	1.57	4.52
Cumulative	0.41	0.69	0.93	4.52

almost identical to the effect of public investment I find. This is despite the fact that my model does not feature amplification effects through aggregate demand. Neither public investment spending itself nor higher consumption demand of workers due to improved labor market conditions stimulate aggregate output. If output was partially demand determined instead, the employment multiplier of public investment would likely be even larger. Indeed, the interest rate in an extended model with a consumption savings decision of workers (see online Appendix B.3) increases in response to the expansion in public investment which indicates an increase in aggregate private consumption demand.

The employment multiplier of unproductive spending (i.e., government consumption) is zero in the baseline scenario, in which spending is financed with lump-sum taxes on firm owners, because government spending crowds out consumption of firm owners one for one. Hence, the multiplier of public investment here equals the extent to which the employment effect of public investment exceeds the effect of unproductive government consumption. Interpreted this way, an excess multiplier of public investment over government consumption of 2.49 jobs per million dollars is large.

Table 4 shows, at different horizons, the output multipliers associated with the expansion of public investment. The first row displays the peak multiplier, the maximum change in output divided by the change in public investment over the respective horizon, $\max_{h \leq H} \Delta Y_h / \Delta I_h^G$. The second row is the cumulative multiplier, the cumulative output change divided by the cumulative change in public investment, i.e., $\sum_{h \leq H} \Delta Y_h / \sum_{h \leq H} \Delta I_h^G$. The table shows that the anticipation effect of public investment, without additional amplification through aggregate demand, already leads to output multipliers in the range of empirical estimates of overall government spending multipliers (Ramey 2011).

Multipliers increase over time, as both the employment effects and the productivity effects of public investment take time to materialize. This is also why the peak multipliers in the short run are generally larger than the cumulative multipliers.

It is instructive to compare the short-run responses to the long-run effect of the increase in public investment shown in Figure 3. As stated

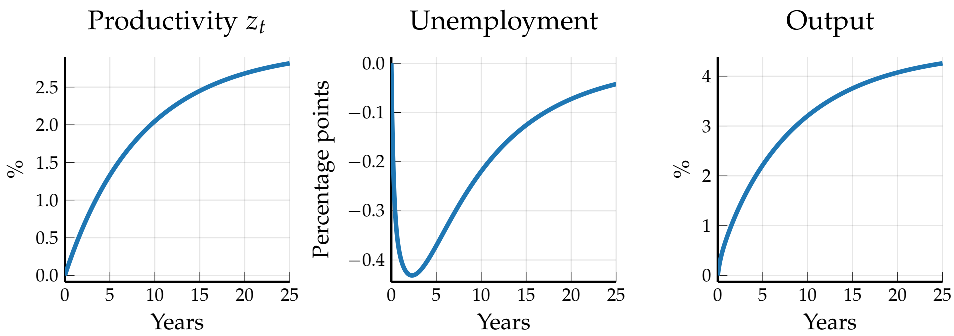


Figure 3: Long-run responses to expansion of government investment
Notes: Responses to a permanent expansion of public investment by 1% of initial GDP as deviations from the initial steady state.

in proposition 1, the investment program does not affect unemployment in the long run. This is because, in the long run, vacancy posting costs, wages, and unemployment benefits are all proportional to labor productivity. As a result, higher productivity does not affect firms' incentives to post vacancies in the long run. Moreover, since workers have logarithmic utility, the constant wage replacement rate of unemployment benefits implies that workers' search effort is unaltered in the long-run. Importantly, unemployment falls below its long-run level temporarily and reaches its trough after 2.5 years. The reason for this is twofold. First, wage inertia implies that wages take time to catch up to increased productivity. This temporarily raises the share of the match surplus received by firms who respond by expanding vacancy creation. Second, vacancy posting costs only depend on the current level of labor productivity whereas the value of a filled vacancy to a firm also depends on future productivity. Therefore, when productivity grows, the surplus is large relative to the costs of creating a vacancy which leads to an expansion in vacancy creation and low unemployment. As growth in labor productivity returns to its long-run trend, the difference between match surplus and vacancy posting costs declines, firms post fewer vacancies and unemployment increases.

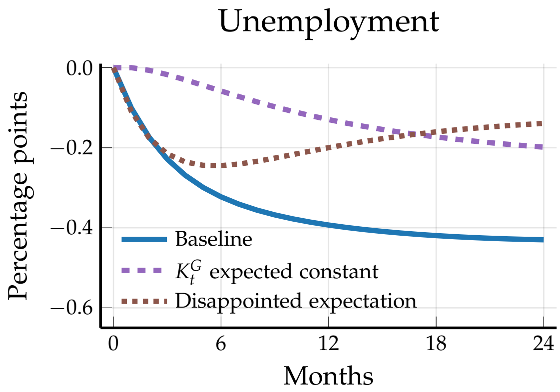
So far, I have considered a permanent expansion in public investment. One might be concerned that the employment effects of a public investment program that is not permanent are substantially smaller. However, the short-run responses to an expansion of public investment by one percent of GDP over only 5 years is almost the same as for a permanent expansion in public investment (see Figure 13 in the online Appendix).

5.1 Implementation delays

We now consider implementation delays which are of interest for two reasons. First, the existing literature has emphasized delays as an important characteristic of government investment, which sets it apart from consumptive government spending and which can impair its effectiveness as a means of short term stimulus (Leeper et al. 2010). Second, comparing how the economy responds to the investment program under different implementation delays allows us to better understand the mechanism through which it affects the economy in the short-run. In particular, it helps to disentangle the expectations effect from the consequences of the contemporaneous increase in productivity.

The dashed red lines in Figure 2 show the responses when it takes six months after the announcement of the investment program before it is implemented and starts to have an effect on productivity. The dotted green lines correspond to the case where the delay amounts to twelve months. In both cases, output and unemployment respond already upon announcement of the investment program.

With an implementation delay of six months, unemployment is almost 0.35 percentage points lower twelve months after the announcement. This is more than three quarters of the decline without the delay. Similarly, output after one year is close to 0.6% higher than without the expansion in public investment. If the delay amounts to twelve months, the investment program still reduces unemployment after one year by about 0.22 percentage points, more than half the reduction without any



Notes: Dashed purple line: agents only find out about increases in productivity as they materialize and expect constant productivity at every point in time. Dotted brown line: agents always expect the increase in productivity to start in the next period, but it never happens.

Figure 4: The anticipation effect on unemployment.

delay. Output after twelve months is still close to 0.3% higher. Importantly, the increase in output and decline in unemployment take place before the investment program has had any effect on productivity (see the top left panel of Figure 2). The observed effect is entirely due to agents anticipating higher productivity in the future as a result of more government investment.

5.2 The anticipation effect

To quantify the contribution of the *anticipated* increase in future productivity to the reduction in unemployment, I consider the following hypothetical scenario. I assume that private agents do not learn about the permanent expansion in public investment in period zero. Instead, they expect productivity to stay constant at every point in time. In period zero, they expect productivity to stay at its steady state level forever. In period one, they are surprised that productivity has increased but expect it to stay at the new level such that in period two they are surprised again by the additional increase. In short, agents only learn about increases in productivity as they occur. The dashed purple line in Figure 4 shows the evolution of unemployment in this case. It still declines but more slowly than when the anticipation effect is present. After one year, unemployment has fallen by 0.13 percentage points, more than 65% less than in the baseline scenario. I interpret this difference as the contribution of the anticipation effect to the unemployment reduction.

An alternative way to quantify the anticipation effect is to consider the case where private agents expect the permanent expansion in public investment to begin at every point in time even though this is never the case. In other words, agents anticipate a permanent expansion of public investment in period zero and act accordingly. They are surprised in period one that productivity has not increased but believe that the increase is going to start in the next period when they are disappointed again. I could then interpret the change in unemployment under this scenario as the contribution of the anticipation effect to the overall reduction in unemployment. It is depicted by the dotted brown line in Figure 4. Initially, the response is identical to the one in the baseline scenario. The two then diverge since wages continue to rise as workers keep bargaining for higher wages in anticipation of increasing productivity even though this increase never materializes. After one year, unemployment has declined by 0.18 percentage points under this scenario. This amounts to 45% of the reduction in the baseline scenario. Accordingly, I would attribute 45% of the unemployment reduction to the anticipation effect.

For both definitions, the anticipation effect accounts for a large part of the reduction in unemployment in response to the expansion in public investment.

5.3 Labor supply response

Higher future productivity due to the announcement of the public investment program affects not only firms' labor demand but also the behavior of workers, the supply side of the labor market. Two changes in workers' behavior are important. First, workers demand a higher wage, since higher future productivity increases the expected total surplus from the match. The resulting wage increase depends on workers' bargaining weight and the degree of wage inertia. Figure 5 shows that the news about higher future productivity raise the Nash-bargained wage substantially, by 3% on impact. Due to wage inertia, the increase in Nash wages only gradually translates into actual wage gains and the actual wage increases almost linearly during the first years after the

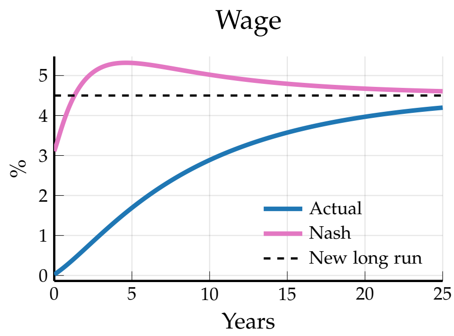


Figure 5: Responses of actual and Nash bargaining wage.

start of the investment program.

Second, workers respond to the anticipated increase in productivity by adjusting their search effort. To assess the importance of workers' search effort (labor supply), I decompose the change in the job-finding probability in every period according to

$$\pi_t^{e|u} - \bar{\pi}^{e|u} = \underbrace{\frac{\pi_t^{e|u}}{\ell_t(u)}\bar{\ell}(u) - \frac{\bar{\pi}^{e|u}}{\bar{\ell}(u)}\bar{\ell}(u)}_{\text{vacancy posting (labor demand)}} + \underbrace{\frac{\pi_t^{e|u}}{\ell_t(u)}\ell_t(u) - \frac{\pi_t^{e|u}}{\ell_t(u)}\bar{\ell}(u)}_{\text{search effort (labor supply)}}, \quad (30)$$

where a bar denotes the variable in the initial steady state. This is the numerical implementation of equation (25) in section 3. The left panel of Figure 6 shows this decomposition graphically. The purple area is the part due to changes in search effort, the terms labeled "search effort" in (30), and the yellow area is the part due to changes in labor demand, the terms labeled "vacancy posting" in (30). The increase in the job-finding probability is almost entirely due to changes in firms' labor demand. Search effort also contributes to the increase, but its effect is negligible. In the first period, the job-finding probability increases by 1.4819 percentage points. Only 0.0005 percentage points are due to the expansion in search effort.

As discussed in the theoretical analysis, two forces drive the response of effort: the expected gain in lifetime utility from finding a job and the marginal effect of higher effort on the current job-finding probability (see equation (26)). The center panel in Figure 6 shows that the expected gain in lifetime utility from finding a job declines in re-

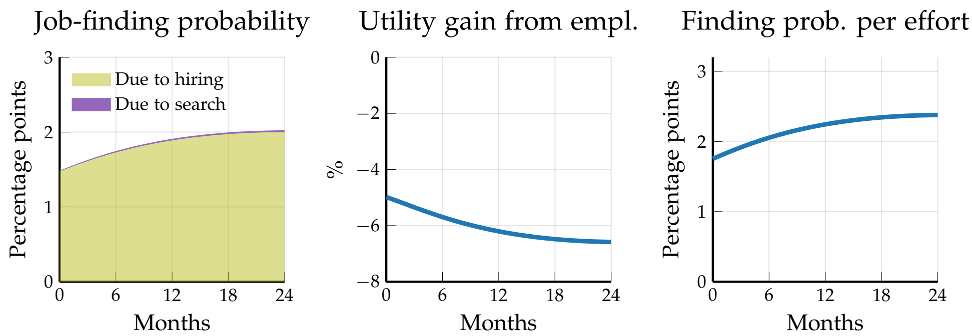


Figure 6: The response of the job-finding probability.

sponse to the investment program. The reason is that job-finding rates increase such that unemployed workers can expect to find employment faster. The drop in the expected gain in lifetime utility from finding a job would lead workers to lower their search effort. However, for the baseline scenario, this effect is dominated by the increase in the job-finding probability per unit of effort depicted in the right panel of Figure 6. Hence, the anticipation effect on short-run labor demand raises employment in two ways. First, it directly increases the probability of finding a job, which in turn raises employment. Second, it indirectly affects employment by inducing workers to expand search effort.

5.4 Financing with distortionary labor taxes

So far, the government financed the investment program with non-distortionary lump-sum taxes on firm owners. Alternatively it could raise the proportional labor tax. Figure 7 shows the responses of key variables when the government raises labor taxes at the same time that expenditures increase. As can be seen from the top left panel, unemployment falls less in response to the program in this case, but it still declines substantially. After one year, it is 0.25 percentage points lower than without the program.

There are two forces that dampen the reduction in unemployment compared to the baseline scenario. First, the increase in the labor tax rate leads to a faster increase in wages as Nash bargaining implies that workers and firms share the tax burden depending on their bargaining

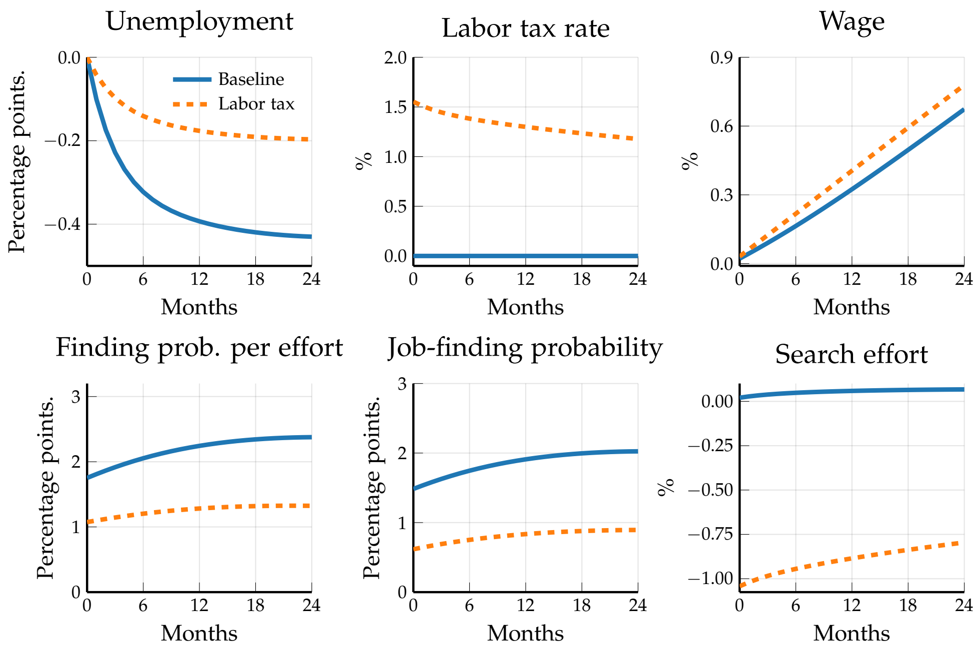


Figure 7: Responses to the investment program financed with proportional labor taxes.

weights. Since wages rise faster, firms do not expand vacancy creation as much as in the baseline. This can be seen from the bottom left panel in Figure 7 which shows that the job-finding probability per unit of search effort increases less if the program is financed with labor taxes. Second, higher labor taxes reduce workers' search effort. It is lower than in the case of lump-sum taxes and actually declines relative to the steady state. There are two reasons for this. First, firms expand vacancy creation less such that the marginal effect of effort on the job-finding probability is lower. Second, the increase in the labor tax reduces the income difference between unemployed and employed workers such that unemployed workers exert less effort.

5.5 Business cycle dependence

Proposition 2 establishes that the size of the employment effect depends on the initial level of unemployment. How large are the differences between boom and recession?

To answer this question, I follow the same approach as in Section 3 and define a recession as an equilibrium with high unemployment and weak labor demand. More specifically, unemployment is 3 percentage points higher than in the steady state and the wage is 2% higher. I define a boom symmetrically, as an equilibrium in which initial unemployment is 3 percentage points lower and the wage 2% higher. The unemployment rate in the recession is thus 9.5 percent, similar to the levels in 2009 to 2010 during the Great Recession. The unemployment rate in the boom is 3.5 percent, close to the rates observed in 2019. I further assume that unemployment benefits are constant at the steady state level. Moreover, the capital to labor ratio is also at the steady state level initially, i.e., the private capital stock is smaller in a recession and higher in a boom.

In the recession, labor market tightness is about twice as large as in the boom. This roughly corresponds to the difference between the trough in tightness at around 0.35 in August 2003 and the peak at 0.73 in March 2007. Comparing the Great Recession to the following expansion, the differences in tightness were even larger. Labor market tightness in 2019 was about 7 times higher than in 2010, 1.2 compared to 0.17.

I study the perfect foresight equilibrium under these differential initial conditions, comparing the case with an expansion in public investment to the one without. Figure 8 shows how unemployment and output respond to the expansion in public investment for the case where the economy is in a recession initially and for the case where it is in a boom. Shown are the deviations from the paths that would be observed without the investment program.[15] When the economy is in a recession initially, the short-run response of both unemployment and output is much larger than when the economy is in a boom. One year after the expansion in public investment, unemployment has fallen by 0.57 percentage points in the case of a recession whereas it has only fallen by 0.4 percentage points in case of a boom. This is a difference of more than 40%.

In online Appendix C.1, I also compare recessions and booms that

15. Figure 14 in the online Appendix shows the evolution of employment without the expansion in public investment.

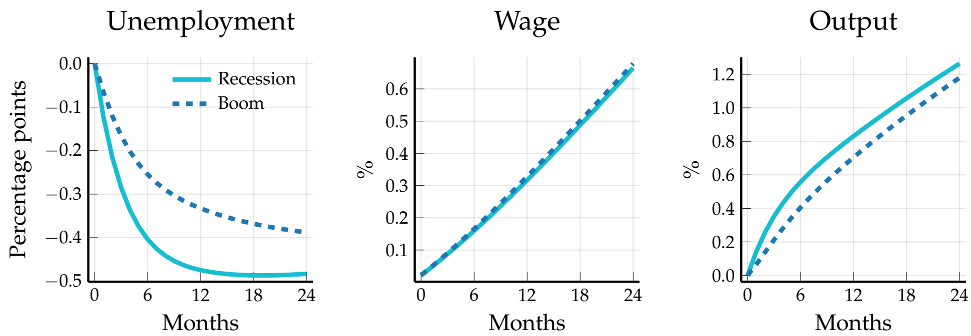


Figure 8: Response of unemployment and output.
Notes: Shown are the deviations from the respective paths that would be observed without the expansion in public investment (see Figure 14).

are the result of shocks to productivity. In this case, the employment effect after one year is about 25% larger when the expansion of public investment is initiated during a recession compared to a boom.

6 Conclusion

Recently, policymakers in several countries proposed plans to expand public infrastructure investment. The hope was that public investment would not only foster long-run economic growth but also provide a stimulus supporting the recovery from the Covid-19 recession. To study whether public investment can provide such stimulus, the existing literature has relied on variants of the neoclassical growth model with frictionless labor markets. In this paper, I revisited this question in a macroeconomic model with search and matching labor market. My theoretical analysis highlighted the role of firms' expectations about future productivity for their hiring decision and the short-run employment effect of public investment. When firms anticipate higher productivity in the future, they expand hiring already in the short run. This mechanism is absent in models without labor market frictions. For a realistic calibration of the model, the anticipation effect is large. It accounts for 65% of the reduction in unemployment by 0.4 percentage points within one year after a permanent expansion of public investment by 1% of GDP. The employment effect is about 40% larger in a recession than in a boom.

These findings are relevant for policymakers. They suggest that a permanent change in fiscal policy towards more public investment can provide a substantial short-run stimulus by raising labor demand. These short-run employment effects are especially large in a recession when labor demand is weak. Thus, a recession might be a good time to initiate a change in fiscal policy towards more public investment. Because much of the short-run employment effects are due to the anticipation effect, the announcement of the policy change already leads to significant employment effects. The exact timing of the implementation is of lesser importance, and credibly announcing the change during a recession is enough to stabilize employment.

In this paper, I provided a positive analysis of the short-run employment effects of public investment. An interesting question is how public investment affects welfare in the short-run. I take some steps in this direction in the online Appendix, where I show that the anticipation effect on labor demand from public investment can improve labor market efficiency if it is inefficiently low.

References

Aschauer, David Alan. 1989. "Does Public Capital Crowd Out Private Capital?" *Journal of Monetary Economics* 24 (2).

Auerbach, Alan J, and Yuriy Gorodnichenko. 2012. "Measuring the Output Responses to Fiscal Policy." *American Economic Journal: Economic Policy* 4 (2): 1–27.

Baxter, Marianne, and Robert G King. 1993. "Fiscal Policy in General Equilibrium." *American Economic Review* 83 (3): 315–334.

Beaudry, Paul, and Franck Portier. 2006. "Stock Prices, News, and Economic Fluctuations." *American Economic Review* 96 (4): 1293–1307.

———. 2007. "When Can Changes in Expectations Cause Business Cycle Fluctuations in Neo-Classical Settings?" *Journal of Economic Theory* 135 (1): 458–477.

Boehm, Christoph E. 2020. "Government Consumption and Investment: Does the Composition of Purchases Affect the Multiplier?" *Journal of Monetary Economics* 115:80–93.

Bom, Pedro R.D., and Jenny E. Ligthart. 2014. "What Have We Learned from Three Decades of Research on the Productivity of Public Capital?" *Journal of Economic Surveys* 28 (5): 889–916.

Bouakez, Hafedh, Michel Guillard, and Jordan Roulleau-Pasdeloup. 2017. "Public Investment, Time to Build, and the Zero Lower Bound." *Review of Economic Dynamics* 23:60–79.

Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg. 2019. "The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective." *The Review of Economic Studies* 87 (1): 77–101.

Chetty, Raj. 2008. "Moral Hazard versus Liquidity and Optimal Unemployment Insurance." *Journal of Political Economy* 116 (2): 173–234.

Chodorow-Reich, Gabriel, Laura Feiveson, Zachary Liscow, and William Gui Woolston. 2012. "Does State Fiscal Relief During Recessions Increase Employment? Evidence from the American Recovery and Reinvestment Act." *American Economic Journal: Economic Policy* 4 (3): 118–145.

Chodorow-Reich, Gabriel, and Loukas Karabarbounis. 2016. "The Cyclicality of the Opportunity Cost of Employment." *Journal of Political Economy* 124 (6): 1563–1618.

Clemens, Marius, Claus Michelsen, and Malte Riethd. 2022. "An Estimation and Decomposition of the Government Investment Multiplier."

Cubas, German. 2020. "Public Capital and Economic Development." *The Economic Journal* 130 (632): 2354–2381.

Den Haan, Wouter J., and Georg Kaltenbrunner. 2009. "Anticipated Growth and Business Cycles in Matching Models." *Journal of Monetary Economics* 56 (3): 309–327.

Den Haan, Wouter J., Garey Ramey, and Joel Watson. 2000. "Job Destruction and Propagation of Shocks." *American Economic Review* 90 (3): 482–498.

Gechert, Sebastian. 2015. "What fiscal policy is most effective? A meta-regression analysis." *Oxford Economic Papers* 67 (3): 553–580.

Gertler, Mark, Christopher Huckfeldt, and Antonella Trigari. 2020. "Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires." *The Review of Economic Studies* 87 (4): 1876–1914.

Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* 98 (4): 1692–1706.

Hall, Robert E. 2003. *Wage Determination and Unemployment Fluctuations.* National Bureau of Economic Research Working Paper 9967.

———. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review* 95 (1): 50–65.

Hyatt, Henry R., and James R. Spletzer. 2016. "The shifting job tenure distribution." SOLE/EALE conference issue 2015, *Labour Economics* 41:363–377.

Kaplan, Greg, Giovanni L. Violante, and Justin Weidner. 2014. "The Wealthy Hand-to-Mouth." *Brookings Papers on Economic Activity* Spring:77–153.

Krusell, Per, Toshihiko Mukoyama, and Ayegül ahin. 2010. "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations." *The Review of Economic Studies* 77 (4): 1477–1507.

Landais, Camille, Pascal Michaillat, and Emmanuel Saez. 2018. "A Macroeconomic Approach to Optimal Unemployment Insurance: Applications." *American Economic Journal: Economic Policy* 10 (2): 182–216.

Leduc, Sylvain, and Daniel Wilson. 2013. "Roads to Prosperity or Bridges to Nowhere? Theory and Evidence on the Impact of Public Infrastructure Investment." *NBER Macroeconomics Annual* 27 (1): 89–142.

Leeper, Eric M., Todd B. Walker, and Shu-Chun S. Yang. 2010. "Government Investment and Fiscal Stimulus." *Journal of Monetary Economics* 57 (8): 1000–1012.

McKay, Alisdair, and Ricardo Reis. 2021. "Optimal Automatic Stabilizers." *The Review of Economic Studies.*

Merz, Monika. 2010. "Comment to Unemployment Fiscal Multiplier by Tommaso Monacelli, Roberto Perotti, and Antonella Trigari." *Journal of Monetary Economics* 57 (5): 554–556.

Michaillat, Pascal. 2014. "A Theory of Countercyclical Government Multiplier." *American Economic Journal: Macroeconomics* 6 (1): 190–217.

Michaillat, Pascal, and Emmanuel Saez. 2018. "Optimal Public Expenditure with Inefficient Unemployment." *The Review of Economic Studies* 86 (3): 1301–1331.

Mitman, Kurt, and Stanislav Rabinovich. 2015. "Optimal Unemployment Insurance in an Equilibrium Business-Cycle Model." *Journal of Monetary Economics* 71:99–118.

Monacelli, Tommaso, Roberto Perotti, and Antonella Trigari. 2010. "Unemployment fiscal multipliers." *Journal of Monetary Economics* 57 (5): 531–553.

Mortensen, Dale T., and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *The Review of Economic Studies* 61 (3): 397–415.

Munnell, Alicia. 1990. "Why Has Productivity Growth Declined? Productivity and Public Investment." *New England Economic Review,* 3–22.

Pereira, Alfredo M., and Rafael Flores de Frutos. 1999. "Public Capital Accumulation and Private Sector Performance." *Journal of Urban Economics* 46 (2): 300–322.

Petrongolo, Barbara, and Christopher A Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 39 (2): 390–431.

Pissarides, Christopher A. 1985. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *The American Economic Review* 75 (4): 676–690.

———. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77 (5): 1339–1369.

Ramey, Valerie A. 2011. "Can Government Purchases Stimulate the Economy?" *Journal of Economic Literature* 49 (3): 673–685.

———. 2020. *The Macroeconomic Consequences of Infrastructure Investment.* National Bureau of Economic Research Working Paper 27625.

Ravn, Morten O, and Vincent Sterk. 2020. "Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach." *Journal of the European Economic Association* 19 (2): 1162–1202.

Rendahl, Pontus. 2016. "Fiscal Policy in an Unemployment Crisis." *The Review of Economic Studies* 83 (3): 1189–1224.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2012. "What's News in Business Cycles." *Econometrica* 80 (6): 2733–2764.

Serrato, Juan Carlos Suárez, and Philippe Wingender. 2016. *Estimating Local Fiscal Multipliers.* National Bureau of Economic Research Working Paper 22425.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95 (1): 25–49.

———. 2010. *Labor Markets and Business Cycles.* Princeton, NJ: Princeton University Press.

———. 2012. "Reassessing the Ins and Outs of Unemployment." *Review of Economic Dynamics* 15 (2): 127–148.

Summers, Lawrence. 2009. *A Weakened Economy: How to Respond?, 110th Congress (Testimony of Lawrence Summers).*

Wilson, Daniel J. 2012. "Fiscal Spending Jobs Multipliers: Evidence from the 2009 American Recovery and Reinvestment Act." *American Economic Journal: Economic Policy* 4 (3): 251–282.

Appendix

Equilibrium Definition

An equilibrium of this economy is defined as follows.

Definition 2 (Equilibrium)**.** *An equilibrium is a collection of individual sequences of workers' effort and consumption* $\{\ell_t(s^t), c_t(s^t))\}_{t=0}^{\infty}$*, of labor market tightness, capital rental rates and wages,* $\{\theta_t, r_t^k, w_t\}_{t=0}^{\infty}$*, aggregate employment, aggregate capital, and capital per match,* $\{N_t, K_t, k_t\}_{t=0}^{\infty}$*, and of policies* $\{T_t, \tau_t, K_t^G, I_t^G\}_{t=0}^{\infty}$*, such that*

1. *the sequences of effort and consumption* $\{(\ell_t(s^t), c_t(s^t)\}_{t=0}^{\infty}$ *solve the worker problem* (10)*,*
2. *firms choose capital optimally according to* (5)*,*
3. *the sequence of labor market tightness* $\{\theta_t\}_{t=0}^{\infty}$ *ensures that the value of an open vacancy is zero,* $V_t(\theta_t) = 0$*,*
4. *wages are determined according to* (17) *together with* (18) *or* (19)*,*
5. *firm owners choose capital optimally according to* (16)*,*
6. *the capital market clears* $K_t = k_t N_t$*,*
7. *employment follows the law of motion* (13)*,*
8. *the government budget constraint* (21) *holds, public capital follows the law of motion* (20) *and determines productivity according to* (2)*.*

Proof of Proposition 1

The proof proceeds as follows. First, I derive the effect of the expansion in public investment on the sequence of job-finding probabilities. For part i), I show that the job-finding probability strictly increases. The positive employment effect then follows from the law of motion for employment, (13). For part ii), I show that the change in the job-finding probability goes to zero in the long run, as $t \to \infty$ if wages are not completely sticky. The law of motion for employment then implies that employment is unchanged in the long run.

The job-finding probability is

$$\pi_t^{e|u} = \zeta^{\frac{1}{\eta}} q_t^{v\,\frac{\eta-1}{\eta}}.$$

Without adjustment costs, capital in every match is

$$k_t = \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} z_t^{\frac{1}{1-\alpha}}$$

such that the job-creation equation yields

$$q_t^v = \frac{\kappa a^\alpha z_t^{\frac{1}{1-\alpha}}}{\sum_{s=1}^\infty \beta^s (1-\rho)^{s-1} \left\{(1-\alpha) a^\alpha z_{t+s}^{\frac{1}{1-\alpha}} - w_{t+s}\right\}}$$

with $a \equiv \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}$. Hence, the job-finding probability is

$$\begin{aligned}\pi_t^{e|u} =& \zeta^{\frac{1}{\eta}} \beta^{\frac{1-\eta}{\eta}} \left(z_t^{\frac{1}{1-\alpha}} \kappa a^\alpha\right)^{\frac{\eta-1}{\eta}} \\ &\times \left[\sum_{s=t}^{\infty} (\beta(1-\rho))^{s-k} \left[(1-\alpha) a^\alpha z_{s+1}^{\frac{1}{1-\alpha}} - w_{s+1}\right]\right]^{\frac{1-\eta}{\eta}}.\end{aligned}$$

The job-finding probability depends on the sequence of private productivity z_s and on the wage sequence w_s. For this reason, we determine next, how private productivity z_s and the wage w_s respond to the marginal increase in investment.

Productivity is

$$z_s = A\left(K_s^G\right)^\vartheta$$

and so

$$\frac{\mathrm{d}z_s}{\mathrm{d}x} = A\vartheta K_s^{G\,\vartheta-1} \frac{\mathrm{d}K_s^G}{\mathrm{d}x} = z_n \frac{\vartheta}{K_s^G} \frac{\mathrm{d}K_s^G}{\mathrm{d}x}.$$

Furthermore,

$$K_s^G = (1-\delta_G)^s K_0^G + \sum_{j=0}^{s-1}(1-\delta_G)^{s-1-j} I_j^G + \sum_{j=T}^{s-1}(1-\delta_G)^{s-1-j} x$$

such that

$$\frac{\mathrm{d}K_s^G}{\mathrm{d}x} = \sum_{j=T}^{s-1}(1-\delta_G)^{s-1-j}$$

and

$$\frac{\mathrm{d}z_s}{\mathrm{d}x} = z_s \frac{\vartheta}{K_s^G} \sum_{j=T}^{s-1}(1-\delta_G)^{s-1-j}.$$

By assumption $I^G = \delta_G K_s^G$ for all s, such that z_s is constant and

$$\frac{\mathrm{d}z_s}{\mathrm{d}x} = \begin{cases} \frac{\vartheta z_s}{I^G}(1-(1-\delta_G)^{s-T}), & \text{if } s > T \\ 0 \quad \text{if } s \leq T. \end{cases}$$

The wage is

$$w_s = \gamma^s w_0 + \sum_{n=1}^{t}(1-\gamma)\omega a^\alpha z_n^{\frac{1}{1-\alpha}} \gamma^{t-n}$$

such that

$$\frac{\mathrm{d}w_s}{\mathrm{d}x} = \sum_{n=1}^{s} \gamma^{s-n}(1-\gamma)\omega a^\alpha \frac{1}{1-\alpha} z_n^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{d}z_n}{\mathrm{d}x}.$$

I distinguish two cases, $t \leq T$ and $t > T$.

Case 1: $t \leq T$ The semi-elasticity of the job-finding probability in period $t \leq T$ with respect to public investment in the periods after T is

$$
\begin{aligned}
\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}|_{x=0} = & \left[\frac{(1-\alpha-\omega)a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} - \frac{\gamma^{t+1}w_0}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha z^{\frac{1}{1-\alpha}}\gamma^{t+1}}{1-\gamma\beta(1-\rho)}\right]^{-1} \\
& \times \pi_t^{e|u}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}z^{\frac{\alpha}{1-\alpha}}\left[\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-t}\right. \\
& \left.\times\left[(1-\alpha)a^\alpha\frac{\partial z_{s+1}}{\partial x} - (1-\gamma)\omega a^\alpha\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\frac{\partial z_n}{\partial x}\right]\right] \\
= & \left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{t+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha\gamma^{t+1}}{1-\gamma\beta(1-\rho)}\right]^{-1} \\
& \times \pi_t^{e|u}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}\frac{\vartheta}{\delta_G K_G}(\beta(1-\rho))^{T-t} \\
& \times\left[(1-\alpha)a^\alpha\left(\frac{1}{1-\beta(1-\rho)} - \frac{1-\delta_G}{1-\beta(1-\delta_G)(1-\rho)}\right)\right. \\
& - (1-\gamma)\omega a^\alpha\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-T} \\
& \left.\times\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\left(1-(1-\delta_G)^{n-T}\right)\right] \\
= & \left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{t+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha\gamma^{t+1}}{1-\gamma\beta(1-\rho)}\right]^{-1} \\
& \times \pi_t^{e|u}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}\vartheta(\beta(1-\rho))^{T-t} \\
& \times\left[(1-\alpha)a^\alpha\frac{\delta_G}{(1-\beta(1-\rho))(1-\beta(1-\delta_G)(1-\rho))}\right. \\
& - (1-\gamma)\omega a^\alpha\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-T} \\
& \left.\times\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\left(1-(1-\delta_G)^{n-T}\right)\right]\frac{1}{I^G}.
\end{aligned}
$$

For the last summand, it holds that

$$\sum_{n=T+1}^{s+1} \gamma^{s+1-n} \left(1-(1-\delta_G)^{n-T}\right)$$
$$= \left(\frac{\gamma^{s-T+1}-1}{\gamma-1} - (1-\delta_G)\frac{\gamma^{s-T+1}-(1-\delta_G)^{s-T+1}}{\gamma-1+\delta}\right)$$

such that

$$\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-t}\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\left(1-(1-\delta_G)^{n-t}\right)$$
$$=\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-t}\Big(\frac{\gamma^{s-T+1}-1}{\gamma-1}-(1-\delta_G)\frac{\gamma^{s-T+1}-(1-\delta_G)^{s-T+1}}{\gamma-1+\delta}\Big)$$
$$=\left(\frac{1}{1-\beta\gamma(1-\rho)}\left(\frac{\gamma}{\gamma-1}-\frac{(1-\delta_G)\gamma}{\gamma-1+\delta_G}\right)-\frac{1}{\gamma-1}\frac{1}{1-\beta(1-\rho)}\right.$$
$$\left.+(1-\delta_G)^2\frac{1}{\gamma-1+\delta_G}\frac{1}{1-\beta(1-\rho)(1-\delta_G)}\right)$$
$$=\frac{1}{(1-\gamma)(1-\gamma-\delta_G)}$$
$$\times\left(\frac{\delta_G\gamma^2}{1-\beta\gamma(1-\rho)}+\frac{1-\gamma-\delta_G}{1-\beta(1-\rho)}-\frac{(1-\gamma)(1-\delta_G)^2}{1-\beta(1-\rho)(1-\delta_G)}\right)$$

and so

$$\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}|_{x=1}=\pi_t^{e|u}\left[\frac{(1-\alpha-\omega)a^{\alpha}}{1-\beta(1-\rho)}-\frac{\gamma^{t+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)}+\frac{\omega a^{\alpha}\gamma^{t+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}$$
$$\vartheta(\beta(1-\rho))^{T-t}\left[a^{\alpha}\frac{\delta_G(1-\alpha-\omega)}{(1-\beta(1-\rho))(1-\beta(1-\delta_G)(1-\rho))}\right.$$
$$\left.+\frac{\gamma\omega a^{\alpha}\delta_G}{(1-\beta\gamma(1-\rho))(1-\beta(1-\rho)(1-\delta_G))}\right]\frac{1}{I^G}\frac{1-\eta}{\eta}\frac{1}{1-\alpha} \tag{31}$$

Case 2: $t > T$

$$
\begin{aligned}
\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}\Big|_{x=0} = & \left[z^{\frac{1}{1-\alpha}} \frac{(1-\alpha-\omega)a^{\alpha}}{1-\beta(1-\rho)} - \frac{\gamma^{t+1} w_0}{1-\beta\gamma(1-\rho)} + \frac{\omega a^{\alpha} z^{\frac{1}{1-\alpha}} \gamma^{t+1}}{1-\gamma\beta(1-\rho)} \right]^{-1} \\
& \pi_t^{e|u} \frac{1-\eta}{\eta} \frac{1}{1-\alpha} z^{\frac{\alpha}{1-\alpha}} \left[\sum_{s=t}^{\infty} (\beta(1-\rho))^{s-t} \left[(1-\alpha) a^{\alpha} \frac{\partial z_{s+1}}{\partial x} \right.\right. \\
& \left.\left. - (1-\gamma)\omega a^{\alpha} \sum_{n=T+1}^{s+1} \gamma^{s+1-n} \frac{\partial z_n}{\partial x} \right]\right] \\
& + \pi_t^{e|u} \vartheta \frac{\eta-1}{\eta} \frac{1}{1-\alpha} \left(1 - (1-\delta_G)^{t-T} \right) \\
= & \left\{ \left[\frac{(1-\alpha-\omega)a^{\alpha}}{1-\beta(1-\rho)} - \frac{\gamma^{t+1} w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^{\alpha} \gamma^{t+1}}{1-\gamma\beta(1-\rho)} \right]^{-1} \right. \\
& \left[(1-\alpha-\omega) a^{\alpha} \left(\frac{1}{1-\beta(1-\rho)} - \frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\rho)(1-\delta_G)} \right) \right. \\
& \left. + \frac{\gamma\omega\delta_G a^{\alpha}}{1-\gamma-\delta_G} \left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)} \right) \right] \\
& \left. - (1 - (1-\delta_G)^{t-T}) \right\} \frac{(1-\eta)\pi_t^{e|u}}{(1-\alpha)\eta} \frac{\vartheta}{I^G}
\end{aligned}
\qquad (32)
$$

With $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$, I get

$$\begin{aligned}
\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}|_{x=0} =&\frac{(1-\eta)\pi_t^{e|u}}{(1-\alpha)\eta}\frac{\vartheta}{I^G}\Bigg\{\left(1-(1-\delta_G)^{t-T+1}\frac{1-\beta(1-\rho)}{1-\beta(1-\rho)(1-\delta_G)}\right)\\
&-(1-(1-\delta_G)^{t-T})+\frac{(1-\beta(1-\rho))\gamma\omega\delta_G}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\\
&\times\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right)\Bigg\}\\
=&\frac{(1-\eta)\pi_t^{e|u}}{(1-\alpha)\eta}\frac{\vartheta}{I^G}\Bigg\{\frac{(1-\delta_G)^{t-T+1}\delta_G}{1-\beta(1-\rho)(1-\delta_G)}\\
&+\frac{(1-\beta(1-\rho))\gamma\omega\delta_G}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\\
&\times\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right)\Bigg\}
\end{aligned}$$

For the proof of part i), note that in both cases $\frac{\partial\pi_t^{e|u}}{\partial x}|_{x=0} > 0$ (assuming that the job-finding probability in the initial equilibrium is strictly positive). The result follows directly for the case $t \leq T$ since $\pi_t^{e|u} > 0$ only if

$$\left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)}-\frac{\gamma^{t+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)}+\frac{\omega a^\alpha\gamma^{t+1}}{1-\gamma\beta(1-\rho)}\right]^{-1} > 0.$$

For the case $t > T$, the crucial step is to note that the wage stickiness term is always positive,

$$\frac{\gamma\omega\delta_G a^\alpha}{1-\gamma-\delta_G}\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right) > 0.$$

This is the case since

$$\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right) > 1-\delta_G-\gamma$$

if and only if $\gamma < 1 - \delta_G$. In this case, $1 - \delta_G - \gamma > 0$ such that also

$$\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right) > 0$$

and thus

$$\frac{\gamma\omega\delta_G a^\alpha}{1-\gamma-\delta_G}\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right) > 0.$$

If in contrast $\gamma > 1 - \delta_G$, then $1 - \delta_G - \gamma < 0$ and

$$\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right) < 0$$

such that also in this case

$$\frac{\gamma\omega\delta_G a^\alpha}{1-\gamma-\delta_G}\left(\frac{(1-\delta_G)^{t-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{t-T+1}}{1-\beta\gamma(1-\rho)}\right) > 0.$$

I have shown that $\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}|_{x=0} > 0$ for all t if the initial wage is $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$. By induction, it then follows from the law of motion for employment, that

$$\frac{\mathrm{d}N_t}{\mathrm{d}x} > 0$$

which proves part (i) of proposition 1 for the case where the initial wage is $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$.

It can be seen from (31) and (32) that for given $\pi_t^{e|u}$, $\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}|_{x=0}$ is weakly increasing in w_0. Hence, as long as $\pi_t^{e|u} > 0$, the statement in (i) also holds if $w_0 > \omega a^\alpha z^{\frac{1}{1-\alpha}}$.

To prove part (ii), observe that for $t \to \infty$ we have $t > T$ and since $\lim_{t\to\infty} \frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}|_{x=0} = 0$, it follows from the law of motion for employment that

$$\lim_{t\to\infty} \frac{\mathrm{d}N_t}{\mathrm{d}x} = 0.$$

Proof of Proposition 2

For the case $k \leq T$, we get from above with $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$,

$$\begin{aligned}
\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} =& \frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta}\frac{\vartheta}{I^G}(\beta(1-\rho))^{T-k}\left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)}\right]^{-1} \\
& \left[(1-\alpha)a^\alpha\left(\frac{1}{1-\beta(1-\rho)}-\frac{1-\delta_G}{1-\beta(1-\delta_G)(1-\rho))}\right)\right. \\
& -\frac{\omega a^\alpha}{(1-\gamma-\delta_G)}\left(\frac{\delta_G\gamma^2}{1-\beta\gamma(1-\rho)}\right. \\
& \left.+\frac{1-\gamma-\delta_G}{1-\beta(1-\rho)}-\frac{(1-\gamma)(1-\delta_G)^2}{1-\beta(1-\rho)(1-\delta_G)}\right) \\
& \left.+\frac{\gamma\omega a^\alpha\delta_G}{(1-\beta\gamma(1-\rho))(1-\beta(1-\rho)(1-\delta_G))}\right] \\
=& \frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta}\frac{\delta_G\vartheta(\beta(1-\rho))^{T-k}}{1-\beta(1-\delta_G)(1-\rho)} \\
& \times\left[1+\frac{\gamma\omega(1-\beta(1-\rho))}{(1-\beta\gamma(1-\rho))(1-\alpha-\omega)}\right]\frac{1}{I^G}.
\end{aligned}$$

If the economy is at the steady state initially, then the employment multiplier is

$$\begin{aligned}
M_t^{Inv}(T,\mathcal{X}_0,\mathcal{I}^G) =& \sum_{k=0}^{t-1}(1-\rho-\pi^{e|u})^{t-k-1}(1-N)\frac{\partial\pi_k^{e|u}}{\partial x} \\
=& \frac{(\beta(1-\rho))^{T+1-t}(1-N)\vartheta}{1-\beta(1-\delta_G)(1-\rho)}\frac{\pi^{e|u}}{K^G}\frac{1}{1-\alpha}\frac{1-\eta}{\eta} \\
& \times\frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^t}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))} \\
& \times\left[1+\frac{\gamma\omega(1-\beta(1-\rho))}{(1-\beta\gamma(1-\rho))(1-\alpha-\omega)}\right].
\end{aligned}$$

Proof of Proposition 3

Part i) follows by induction, from the fact that $\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0}$ is independent of the initial level of unemployment and strictly positive, together with the law of motion for employment,

$$N_{t+1} = (1-\rho)N_t + \pi_t^{e|u} U_t = (1-\rho-\pi_t^{e|u})(1-U_t) + \pi_t^{e|u}.$$

Take two initial levels of unemployment, $\tilde{U}_0$ and U_0. Suppose $\tilde{U}_0 > U_0$, then, since $1-\rho > \pi_t^{e|u}$ for all t, $\tilde{U}_1 > U_1$. Moreover, if $\tilde{U}_t > U_t$, then $\tilde{U}_{t+1} > U_{t+1}$. Hence, $\tilde{U}_t > U_t$ for all t. Taking the derivative of the law of motion yields

$$\frac{\partial N_{t+1}}{\partial x} = (1-\rho-\pi_t^{e|u})\frac{\partial N_t}{\partial x} + \frac{\partial \pi_t^{e|u}}{\partial x} U_t.$$

Hence, $\frac{\partial \tilde{N}_1}{\partial x} > \frac{\partial N_1}{\partial x}$. In addition, $\frac{\partial \tilde{N}_{t+1}}{\partial x} > \frac{\partial N_{t+1}}{\partial x}$ if $\frac{\partial \tilde{N}_t}{\partial x} > \frac{\partial N_t}{\partial x}$. It follows that $\frac{\partial \tilde{N}_t}{\partial x} > \frac{\partial N_t}{\partial x}$ for all t.

To prove part ii), I show that the change in the job-finding probability in every period is increasing in w_0 if $\eta > 0.5$. We have from above, that

$$\begin{aligned}\frac{\partial^2 \pi_t^{e|u}}{\partial x \partial w_0} =& \frac{\partial \pi_t^{e|u}}{\partial w_0}\frac{1}{\pi_t^{e|u}}\frac{\partial \pi_t^{e|u}}{\partial x} + \frac{\partial \pi_t^{e|u}}{\partial x}\frac{\gamma^{t+1} z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)} \\ &\times \left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{t+1} w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha \gamma^{t+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}.\end{aligned}$$

We have that

$$\left.\frac{\partial \pi_t^{e|u}}{\partial w_0}\frac{1}{\pi_t^{e|u}}\right|_{x=0, w_0=\omega a^\alpha z^{\frac{1}{1-\alpha}}} = \frac{\eta-1}{\eta}\frac{1-\beta\gamma(1-\rho)}{(1-\alpha-\omega)a^\alpha z^{\frac{1}{1-\alpha}}}\frac{\gamma^{t+1} z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)}$$

such that

$$\left.\frac{\partial^2 \pi_t^{e|u}}{\partial x \partial w_0}\right|_{x=0, w_0=\omega a^\alpha z^{\frac{1}{1-\alpha}}} = \frac{2\eta-1}{\eta}\frac{1-\beta\gamma(1-\rho)}{(1-\alpha-\omega)a^\alpha z^{\frac{1}{1-\alpha}}}\frac{\gamma^{t+1} z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)}\frac{\partial \pi_t^{e|u}}{\partial x},$$

which is positive if $\eta > 0.5$.

Proof of Proposition 4

First observe, that the job-finding probability is linear in effort, such that $\frac{\partial \pi_t^{e|u}(\theta_t,\ell_t)}{\partial \ell_t} = \frac{\pi_t^{e|u}(\theta_t,\ell_t)}{\ell_t}$ is independent of ℓ_t. The first-order condition for the effort choice is (12). Application of the implicit function theorem yields

$$\frac{\mathrm{d}\ell_t}{\mathrm{d}x} = \beta \left(\frac{\mathrm{d}\Delta_{t+1}^{eu}}{\mathrm{d}x} \frac{\pi_t^{e|u}}{\ell_t} + \frac{\partial \pi_t^{e|u}}{\partial \theta_t} \frac{\mathrm{d}\theta_t}{\mathrm{d}x} \frac{\Delta_{t+1}^{e|u}}{\ell_t} \right) \frac{1}{\frac{\partial^2 d(\ell_t,u)}{\partial \ell_t}}.$$

Note that $\frac{\partial \Delta_{t+1}^{eu}}{\partial \ell_t} = 0$ by the Envelope Theorem, and $\frac{\partial\left(\pi_t^{e|u}/\ell_t\right)}{\partial \ell_t} = 0$ since the job finding probability is linear in effort. Using the first order condition for effort then gives equation (26) in the main text.

The overall change in the job finding probability is thus

$$\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x} = \frac{\partial \pi_t^{e|u}}{\partial \theta_t} \frac{\mathrm{d}\theta_t}{\mathrm{d}x} + \left(\frac{\partial \pi_t}{\partial \theta_t} \frac{\mathrm{d}\theta_t}{\mathrm{d}x} \frac{1}{\ell_t} + \frac{\pi_t}{\ell_t} \frac{\mathrm{d}\Delta_{t+1}^{eu}}{\mathrm{d}x} \frac{1}{\Delta_{t+1}^{eu}} \right) \frac{\frac{\partial d(\ell_t,u)}{\partial \ell_t}}{\frac{\partial^2 d(\ell_t,u)}{\partial \ell_t^2}}.$$

We want to show that $\frac{\mathrm{d}\pi_t^{e|u}}{\mathrm{d}x}$ can take on positive and negative values. Suppose unemployment benefits are proportional to labor income, such that $\frac{c_t^e}{c_t^u} = \bar{b}$. The difference in lifetime utility between employed and unemployed workers is

$$\Delta_{t+1}^{eu} = \log(\bar{b}) - d(0,e) + d(\ell_t,u) + \beta \left(1 - \rho - \pi_{t+1}^{e|u}\right) \Delta_{t+1}^{e|u},$$

where ℓ_t is the optimal effort choice. Since ℓ_t is optimal, its marginal effect on Δ_{t+1}^{eu} is zero (Envelope Theorem). Thus,

$$\frac{\mathrm{d}\Delta_{t+1}^{eu}}{\mathrm{d}x} = -\beta \frac{\partial \pi_{t+1}^{e|u}}{\partial \theta_{t+1}} \frac{\mathrm{d}\theta_{t+1}}{\mathrm{d}x} \Delta_{t+1}^{eu} + \beta \left(1 - \rho - \pi_{t+1}^{e|u}\right) \frac{\mathrm{d}\Delta_{t+2}^{e|u}}{\mathrm{d}x},$$

and so

$$\frac{\mathrm{d}\Delta_{t+1}^{eu}}{\mathrm{d}x} = \beta \sum_{s=0}^{\infty} \prod_{n=0}^{s} \left(1 - \rho - \pi_{t+n}^{e|u}\right) \beta^s \frac{\partial \pi_{t+1+s}}{\partial \theta_{t+1+s}} \frac{\mathrm{d}\theta_{t+1+s}}{\mathrm{d}x} \Delta_{t+1+s}^{eu}.$$

In the proof of Proposition 1, we derived the labor demand effect on the job-finding probability, $\frac{\partial \pi_{t+1}^{e|u}}{\partial \theta_{t+1}} \frac{\mathrm{d}\theta_{t+1}}{\mathrm{d}x}$.[16] Using this, we find that, at the steady state and without wage stickiness ($\gamma = 0$), the change in the job-finding probability is proportional to

$$\frac{\mathrm{d}\Delta_{t+1}^{eu}}{\mathrm{d}x} \propto (1+\nu)\left(\beta(1-\rho)\right)^{T-t} - \nu\left(\beta(1-\rho)\right)^{T-t} \frac{1-\left(\frac{1-\rho-\pi^{e|u}}{1-\rho}\right)^{T-t}}{\pi^{e|u}}$$
$$-\nu\beta(1-\delta_G)^{t+2-T} \frac{1}{1-\beta(1-\rho-\pi^{e|u})(1-\delta_G)},$$

where $\nu = \frac{\frac{\partial d(\ell_t,u)}{\partial \ell_t}}{\frac{\partial^2 d(\ell_t,u)}{\partial \ell_t^2}\ell_t}$, which is a constant, e.g., $\nu = \chi$ for $d(\ell,u) = d_1 \frac{\ell^1+\chi}{1+\chi} + d_{0,u}$ as considered in the quantitative analysis.

Suppose ν is positive, and vacancy posting costs are high, such that the steady state job-finding probability ($\pi^{e|u}$) is sufficiently small, then the change in the job-finding probability is negative. In contrast, if posting costs are small such that the steady state job-finding probability is sufficiently large, then the change in the job-finding probability is positive. This completes the proof.

16. There, we denoted it simply as $\frac{\mathrm{d}\pi_{t+1}^{e|u}}{\mathrm{d}x}$, because we only the demand effect was present.

Online Appendix

to "The Short-Run Employment Effects of Public Infrastructure Investment"

A Calibration Details

A.1 Estimation of job-finding and separation probabilities

The data source most commonly used to estimate transition rates between labor market states is the Current Population Survey (CPS). There are two main method to estimating the job-finding rate from CPS data. Here, I use the one based on gross flows, that is, I use the panel dimension of the monthly CPS microdata to estimate the number of workers who transition from unemployment to employment in a given month. The alternative approach uses only the aggregate time series of unemployment as described in Shimer (2012). It requires stronger assumptions than the gross flows method used here, in particular, it assumes a constant labor force. In contrast, the gross flows approach can be extended to incorporate more than two labor market states and arbitrary transitions between them. A discussion and comparison of the two methods can be found in Shimer (2012).

I consider two different definitions of unemployed workers, denoted U-3 and U-5 by the BLS. The most widely used concept is U-3. According to this definition a worker is unemployed if i) he or she does not work but has been actively looking for a job during the last four weeks and would be available to work or if ii) he or she is temporarily laid off and waiting to be recalled. The alternative definition, U-5, also encompasses workers who want a job, searched for a job at some point during the last twelve months, and could have taken a job in the last week if they had been offered one. Hence, this measure includes discouraged and marginally attached workers according to the BLS classification.

Figure 9 shows the number of unemployed workers according to the definitions U-3 and U-5 over time.

Following Shimer (2012), I estimate the job-finding probability from gross flows as follows:

- I match individuals across monthly CPS waves from January 1976 to December 2020 to obtain a panel data set
- For every month I compute the number of workers who transition between each of the three labor market states employed, unemployed, inactive
  - I do this for both concepts of unemployment, U-3 and U-5
  - The series are seasonally adjusted using X13-ARIMA-SEATS
- From these flows I obtain a Markov matrix for the monthly transition between the three states for every month in the sample
- I adjust for time aggregation using the method described in Shimer (2012)
  - I compute the continuous time Markov matrix (instantaneous transition probabilities) from the discrete time matrix and obtain the monthly transition probabilities from the instantaneous transition rates. The monthly probabilities obtained in this way capture the probability of experiencing a transition between state A and B over the course of one month. This is different from the probability of being in state B in the next month conditional on being in state A in the current month. The latter is what I observe in the data, the former is what I need to inform the calibration of the model.
- To also obtain separate transition probabilities for U-3 unemployed and marginally attached workers, I use the same procedure but with four states (employed, U-3 unemployed, marginally attached, inactive).

Prior to 1994, the CPS did not include the questions used to identify discouraged and marginally attached workers. This is why I can only compute job-finding probabilities of unemployed workers according to the broader definition U-5 for the time period from 1994 to 2020. For comparison, I also compute the transition probabilities according to the unemployment concept U-3 for the whole time period covered by the CPS, January 1976 to December 2020. Table 5 shows the average monthly job-finding probability for U-3 unemployed, U-5 unemployed, and marginally attached workers for different time periods. For the time period from 1994 to 2020, the average job-finding probability for unemployed workers according to the concept U-3 was 29.4%. It was 2.5 percentage points lower for the group of U-5 unemployed workers. Marginally attached workers are much less likely to find a job in a given month, on average their job-finding probability is only 10.9%.

Table 5: Average monthly transition probabilities, 1976–2020 and 1994–2020.

	1976–2020	1994–2020
Job-finding probability U-3	29.8	29.4
Job-finding probability U-5	—	26.9
Job-finding probability marginally attached	—	10.9
Separation rate	1.9	1.8

The reason for the small difference in job-finding probabilities between U-3 and U-5 can be found in Figure 9, which shows the total numbers of unemployed workers according to definitions U-3 and U-5 and the number of marginally attached workers over time. On average, the number of marginally attached workers is only about one fifth of the number of U-3 unemployed workers. For the group of unemployed workers according to the definition U-5, marginally attached workers play a small role. This is why the substantially lower job-finding probability of marginally attached workers does not matter much for the overall job-finding probability in the group of U-5 unemployed workers.

Figure 10 shows the estimated monthly job-finding probability over time. The dark blue line shows the estimated monthly job-finding probability of unemployed workers, when unemployed according to the concept U-3 are considered. For the time period from 1976Q1 to 2007Q2, I

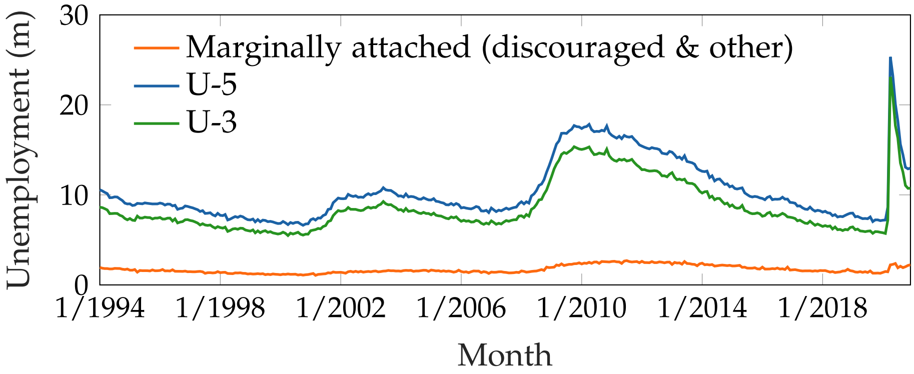


Figure 9: Unemployment in the US, January 1994 to December 2020 (in millions).

can compare the quarterly averages of this series to the series in Shimer (2012). The two are very similar, the standard deviation of the difference is less than 1.5 percentage points. This difference is likely coming from the different seasonal adjustment procedures used. The light blue line represents the job-finding probability for unemployed according to the definition U-5. Finally, the green line shows the job-finding rate for marginally employed workers, when I distinguish between four labor market states, employed, U-3 unemployed, marginally attached, and inactive.

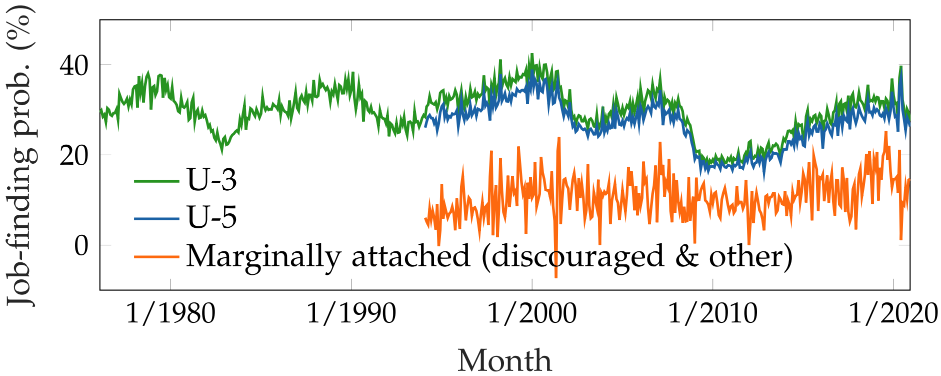


Figure 10: Estimated monthly job-finding probabilities.

A.2 Calibration of disutility from effort

I calibrate the parameter χ to match the elasticity of the job-finding probability with respect to unemployment benefits $\epsilon_{\pi,b} = \frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{b}{\pi^{e|u}}$. From the first-order condition for search effort, I have that

$$\ell^{\chi} = \beta\left(J_t(e) - J_t(u)\right)\frac{\pi^{e|u}}{\ell}. \tag{33}$$

In the steady state the difference between lifetime utility of employed and unemployed workers is

$$J_t(e) - J_t(u) = \frac{\log\left(\frac{w}{b}\right) - d_{0,e} + \frac{\ell^{1+\chi}}{1+\chi}}{1-\beta+\beta(\rho+\pi^{e|u})}$$

Hence

$$\left(1-\beta+\beta(\rho+\pi^{e|u})\right)\ell^{\chi} = \beta\left(\log\left(\frac{w}{b}\right) - d_{0,e} + \frac{\ell^{1+\chi}}{1+\chi}\right)x$$

where $x = \frac{\pi^{e|u}}{\ell}$ is a constant (partial equilibrium) and

$$\left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi-1}\frac{\mathrm{d}\ell}{\mathrm{d}b} + \beta\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\ell^{\chi} = -\beta\frac{1}{b}x + \beta\ell^{\chi}x\frac{\mathrm{d}\ell}{\mathrm{d}x}$$

Since, $\frac{\mathrm{d}\ell}{\mathrm{d}b} = \frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{1}{x}$.

$$\left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi-1}\frac{1}{x}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b} + \beta\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\ell^{\chi} = -\frac{\beta}{b}x + \beta\ell^{\chi}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b} \tag{34}$$

$$\Leftrightarrow \left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{1}{\pi^{e|u}} = -\beta\frac{1}{b}x \tag{35}$$

$$\Leftrightarrow \left(1-\beta+\beta(\rho+\pi^{e|u})\right)\chi\ell^{\chi}\frac{\mathrm{d}\pi^{e|u}}{\mathrm{d}b}\frac{b}{\pi^{e|u}} = -\beta\frac{\pi^{e|u}}{\ell} \tag{36}$$

Substituting (33) for ℓ^{χ} and rearranging yields

$$\chi = -\frac{1}{(1-\beta+\beta(\rho+\pi^{e|u}))\epsilon_{q,b}\left(J_t(e) - J_t(u)\right)}$$

All terms on the right-hand side follow directly from the calibration targets.

B Additional Theoretical Results

B.1 Welfare effects of public investment

The permanent expansion in public investment raises employment as firms expand hiring in anticipation of higher future productivity. I now show that this increase in employment constitutes an efficiency gain when equilibrium labor demand is inefficiently low. In this case, public investment improves labor market efficiency because the anticipation effect stimulates labor demand and brings vacancy creation closer to its efficient level. Therefore, public investment has a positive effect on welfare beyond the return from public investment and redistribution.

I define social welfare as follows

$$W(\{c_t^F, c_t(s^t), \ell_t(s^t)\}) = \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t u^F(c_t^F) + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u\left(c_t(s^t), \ell_t(s^t), s_t\right) \pi_t(s^t|s_0)\bar{\mu}(s_0).$$

Here, $\bar{\mu}^F$, $\bar{\mu}(e)$ and $\bar{\mu}(u)$ are the welfare weights of firm owners, initially employed and initially unemployed workers and $\pi_t(s^t|s_0)$ denotes the share of workers with history $s^t = (s_0, s_1, \ldots, s_t)$ in period t. Let C_t denote aggregate consumption in period t and define the consumption shares of individual firm owners and of workers as $v_t^F \equiv \frac{c_t^F}{C_t}$ and $v_t(s^t) \equiv \frac{c_t(s^t)}{C_t}$.

Under Assumption 1 (fixed search effort), the effect of the investment program on welfare is

$$\frac{\partial W}{\partial x} = \underbrace{\sum_{t=0}^{\infty} \beta^t C_t \left(\bar{\mu}^F u_c^F(c_t^F) \frac{\partial v_t^F}{\partial x} + \sum_{s^t} \bar{\mu}(s_0) \pi_t(s^t|s_0) u_c(c_t(s^t)) \frac{\partial v_t(s^t)}{\partial x} \right)}_{\text{redistribution (intensive margin)}}$$
$$+ \underbrace{\sum_{t=0}^{\infty} \beta^t u(c_t(s^t), \ell_t(s^t), s_t) \bar{\mu}(s_0) \frac{\partial \pi_t(s^t|s_0)}{\partial x}}_{\text{redistribution (extensive margin)}} + \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x}}_{\text{aggregate consumption}}. \tag{37}$$

Here,

$$\mathrm{m}_t \equiv \bar{\mu}^F v_t^F u_c^F(c_t^F) + \sum_{s^t} \bar{\mu}(s_0) \pi_t(s^t|s_0) v_t(s^t) u_c(c_t(s^t)) \tag{38}$$

is the marginal utility of aggregate consumption in period t, a weighted average of individual marginal utilities of consumption, where the weight of each agent corresponds to its welfare weight multiplied by its consumption share.

As can be seen from equation (37), the effect of the expansion in public investment on welfare can be decomposed into three parts. The first captures the effect of public investment on the distribution of consumption along the intensive margin. Depending on how the increase in public investment is financed, consumption of employed workers, unemployed workers or firm owners increases or falls relative to aggregate consumption and this redistribution changes welfare, even if aggregate consumption remains unchanged. This distributive effect is captured by the first line in equation (37). Note that under Assumption 2 wages are independent of taxes such that the government can use labor taxes and lump-sum taxes on firm owners to finance investment in a way that leaves the consumption shares of all households unchanged. In this case there is no redistribution of consumption along the intensive margin and the first line in (37) is zero.

The second effect on welfare emerges because the increase in public investment redistributes consumption (and effort) along the extensive margin as it alters the share of workers who are employed. Proposition 1 showed that employment increases in all periods in response to a permanent expansion in public investment if the wage and public investment are in steady state initially. Hence, the extensive margin redistribution raises welfare for sensible parameter choices under which the after-tax wage exceeds unemployment benefits and compensates for potential utility losses from working.

The last summand in equation (37) captures the welfare effect of changes in aggregate consumption due to a permanent increase in public investment. The change in aggregate consumption is

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t K_t^{\alpha} N_t^{1-\alpha} \frac{\partial z_t}{\partial x}}_{\text{direct gross return}} - \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial I_t^G}{\partial x}}_{\text{costs}} + \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t}_{\text{efficiency gain}} \tag{39}$$

Equation (39) shows that there are three channels through which the permanent increase in public investment affects aggregate consumption. The first two are standard. On the one hand, public investment raises productivity, which leads to an increase in output and consumption. On the other hand, there is a resource cost of public investment that reduces consumption. In the frictional labor market considered here, there is a third channel through which public investment affects output. I label it EG_t for "Efficiency Gain" in equation (39).

If the economy is in the steady state, the efficiency gain is

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t = \frac{1}{1-\eta} \left[w - \eta \left((1-\alpha) z k^{\alpha} + \theta \kappa \right) \right] \sum_{t=0}^{\infty} \beta^t m_t M_{t+1}^{Inv}.$$

It comes from the fact that the equilibrium in the matching labor market is not necessarily efficient such that the employment effect of public investment by itself can improve welfare.[17] When a firm posts a vacancy, it imposes a negative externality on other firms, since the additional vacancy makes it more difficult for other firms to fill theirs. However, there is also a positive externality because every additional vacancy makes it easier for workers to find a job. As shown by Hosios (1990), there exists a wage that internalizes both effects and leads to the optimal level

17. For simplicity, I assume that vacancy posting costs are constant, $\kappa_t = \kappa$. Below, I characterize the effect of public investment on aggregate consumption for the general case in which posting costs can depend on public investment.

of vacancy creation. This wage is such that workers' share of the total match surplus equals the elasticity of the matching function. Here, this is the case if

$$w^* = \eta\left((1-\alpha)zk^\alpha + \theta\kappa\right).$$

I show this formally in Appendix B.2, where I derive the constrained efficient allocation. When the equilibrium wage equals the efficient wage, $w = w^*$, the efficiency gain is zero. In contrast, if the wage exceeds the efficient wage, $w > w^*$, vacancy creation in equilibrium is too low and the expansion in labor demand brought about by the investment program can raise the amount of resources available for consumption. The following proposition summarizes this result.

Proposition 5 (Efficiency gains from public investment)**.** *Suppose the economy is in a steady state with inefficiently low labor demand, $w > w^*$. Then, the public investment program improves labor market efficiency,*

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t > 0.$$

Proof. The welfare function is

$$\begin{aligned} W(\{c_t^F, c_t(s^t), \ell_t(s^t)\}) =& \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t u^F(c_t^F) \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u\left(c_t(s^t), \ell_t(s^t), s_t\right) \pi_t(s^t|s_0)\bar{\mu}(s_0). \end{aligned}$$

We can equivalently express welfare as a function of aggregate consumption and individual consumption shares

$$\begin{aligned} \tilde{W}(\{v_t^F, v_t(s^t), \ell_t(s^t), C_t\}) =& \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t u^F(v_t^F C_t) \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u\left(v_t(s^t)C_t, \ell_t(s^t), s_t\right) \pi_t(s^t|s_0)\bar{\mu}(s_0), \end{aligned}$$

such that

$$
\begin{aligned}
\frac{\partial W}{\partial x} = \frac{\partial \tilde{W}}{\partial x} &= \sum_{t=0}^{\infty} \beta^t \bar{\mu}^F u_c^F(c_t^F) C_t \frac{\partial v_t^F}{\partial x} \\
&+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u_c(c_t(s^t)) C_t \frac{\partial v_t(s^t)}{\partial x} \pi_t(s^t|s_0)\bar{\mu}_0(s_0) \\
&+ \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u(c_t(s^t), \ell_t(s^t), s_t) \frac{\pi_t(s^t|s_0)}{\partial x} \bar{\mu}_0(s_0) \\
&+ \sum_{t=0}^{\infty} \beta^t v_t^F u_c^F(c_t^F) \bar{\mu}^F \frac{\partial C_t}{\partial x} \\
&+ \sum_{t=0}^{\infty} \sum_{s^t} \beta^t v_t(s^t) u_c(c_t(s^t)) \pi_t(s^t|s_0) \bar{\mu}(s_0) \frac{\partial C_t}{\partial x},
\end{aligned}
$$

which yields (37). Furthermore,

$$
C_t = z_t N_t^{1-\alpha} K_t^{\alpha} - \kappa_t \theta_t (1 - N_t) - K_{t+1} + (1 - \delta_k) K_t - I_t^G,
$$

such that

$$
\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} &= \sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \Bigg(N_t^{1-\alpha} K_t^{\alpha} \frac{\partial z_t}{\partial x} + (1-\alpha) z_t N_t^{-\alpha} K_t^{\alpha} \frac{\partial N_t}{\partial x} \\
&+ \alpha z_t N_t^{1-\alpha} K_t^{\alpha-1} \frac{\partial K_t}{\partial x} \\
&+ \kappa_t \theta_t \frac{\partial N_t}{\partial x} - \left(\frac{\partial \kappa_t}{\partial x} \theta_t + \kappa_t \frac{\partial \theta_t}{\partial x} \right) (1 - N_t) \\
&- \frac{\partial K_{t+1}}{\partial x} + (1 - \delta_k) \frac{\partial K_t}{\partial x} - \frac{\partial I_t^G}{\partial x} \Bigg) \\
&= \sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \left(N_t^{1-\alpha} K_t^{\alpha} \frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial x} \right) \\
&+ \sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \Bigg(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k \right) \frac{\partial K_t}{\partial x} \\
&- \frac{\partial K_{t+1}}{\partial x} - \theta_t (1 - N_t) \frac{\partial \kappa_t}{\partial x} \Bigg) \\
&+ \sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \left(\left[(1-\alpha) z_t k_t^{\alpha-1} + \kappa_t \theta_t \right] \frac{\partial N_t}{\partial x} - \kappa_t (1 - N_t) \frac{\partial \theta_t}{\partial x} \right)
\end{aligned}
$$

From the law of motion for employment, we get

$$\kappa_t(1-N_t)\frac{\partial\theta_t}{\partial x} = \left[\frac{\partial N_{t+1}}{\partial x} - (1-\rho-q_t^v(\theta_t)\theta_t)\frac{\partial N_t}{\partial x}\right]\frac{\kappa_t}{(1-\eta)q_t^v(\theta_t)}$$

Using this, we have

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\frac{\partial C_t}{\partial x} = & \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\left(N_t^{1-\alpha}K_t^{\alpha}\frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial .x}\right) \\
& + \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\left(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right)\frac{\partial K_t}{\partial x}\right. \\
& \left. - \frac{\partial K_{t+1}}{\partial x} - \theta_t(1-N_t)\frac{\partial \kappa_t}{\partial x}\right) \\
& + \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t\left(\left[(1-\alpha)z_t k_t^{\alpha-1}\right.\right. \\
& \left. + \kappa_t\theta_t\left(1 + \frac{1-\rho-q_t^v(\theta_t)}{(1-\eta)q_t^v(\theta_t)}\right)\right]\frac{\partial N_t}{\partial x} \\
& \left. - \frac{\kappa_t}{(1-\eta)q_t^v(\theta_t)}\frac{\partial N_{t+1}}{\partial x}\right)
\end{aligned}$$

and with the equilibrium condition

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta\left\{(1-\alpha)z_{t+1}k_{t+1}^{\alpha} - w_{t+1} + (1-\rho)\frac{\kappa_{t+1}}{q_t^v(\theta_{t+1})}\right\}$$

we get

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = &\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left(N_t^{1-\alpha} K_t^{\alpha} \frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial .x} \right) \\
&+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \Bigg(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right) \frac{\partial K_t}{\partial x} \\
&- \frac{\partial K_{t+1}}{\partial x} + \theta_t (1 - N_t) \frac{\partial \kappa_t}{\partial x} \Bigg) \\
&+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left([(1-\alpha) k_t^{\alpha} z_t + \kappa_t \theta_t] \frac{\partial N_t}{\partial x} \right) \\
&- \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\kappa_t \theta_t}{1-\eta} \frac{\partial N_t}{\partial x} \\
&- \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \beta \Bigg(\frac{(1-\alpha) k_{t+1}^{\alpha} z_{t+1} - w_{t+1}}{1-\eta} \\
&+ \frac{(1-\rho)\kappa_{t+1}}{(1-\eta) q_t^v(\theta_{t+1})} \Bigg) \frac{\partial N_{t+1}}{\partial x} \\
&+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{(1-\rho)\kappa_t}{(1-\eta) q_t^v(\theta_t)} \frac{\partial N_t}{\partial x}
\end{aligned}
\tag{40}
$$

Suppose the economy is in a steady state, then the average marginal utility of consumption $\mathrm{m}_t = v^F \bar{\mu}_0 + \frac{1}{C}\sum_{s_0} \bar{\mu}(s_0)$. Then, since $\frac{\partial N_0}{\partial x} = 0$,

$$
\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{(1-\rho)\kappa_t}{(1-\eta) q_t^v(\theta_t)} \frac{\partial N_t}{\partial x} = \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{(1-\rho)\kappa_t}{(1-\eta) q_{t+2}^v(\theta_{t+1})} \frac{\partial N_{t+1}}{\partial x}
$$

and the two terms cancel in equation (40). Furthermore, it follows from the optimal capital choice (see (16)) that $\alpha z_t k_t^{\alpha-1} + 1 - \delta_k = \frac{1}{\beta}$. Together with $\frac{\partial K_0}{\partial x} = 0$ this implies

$$
\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left(\left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right) \frac{\partial K_t}{\partial x} - \frac{\partial K_{t+1}}{\partial x} \right) = 0,
$$

which simplifies equation (40) further and yields

$$\sum_{t=0}^{\infty}\beta^t m_t \frac{\partial C_t}{\partial x} = \sum_{t=0}^{\infty}\beta^t m_t\left(N^{1-\alpha}K^{\alpha}\frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial x}\right) - \sum_{t=0}^{\infty}\beta^t m_t \theta(1-N)\frac{\partial \kappa_t}{\partial x}$$
$$+\sum_{t=0}^{\infty}\beta^t m_t \frac{1}{1-\eta}\left[w - \eta\left((1-\alpha)zk^{\alpha}+\theta\kappa\right)\right] M_{t+1}^{Inv}.$$

The second term in the first line are the costs (or benefits) of changing vacancy posting costs. If posting costs are constant, the term drops out. The second line is the efficiency gain as defined in the main text. Since, the employment multiplier $M_{t+1}^{Inv} > 0$ is positive (proposition 1), the efficiency gain is positive if

$$w > \eta\left((1-\alpha)zk^{\alpha}+\theta\kappa\right) = w^*.$$

□

A similar result can be found in the online appendix of Den Haan and Kaltenbrunner (2009) who study a simplified two-period model. They find that news about future productivity can lead to a resource gain when the Hosios condition is violated.

B.2 Optimal allocation

In general, the equilibrium in the search and matching labor market described above is inefficient due to two congestion externalities. When posting a vacancy, a firm does not take into account the negative effect this has on the likelihood of other firms to fill their vacancies. Similarly, firms fail to internalize that every additional vacancy makes it easier for workers to find a job. As a result, the private benefits of posting a vacancy may exceed or fall below the social benefit.

To better understand how these inefficiencies shape the effects of government investment, I analyze the constrained-efficient allocation, which I define as the one that would be chosen by a utilitarian social planner who is constrained by the matching friction and faces the same capital adjustment costs as firm owners. To that end, I define social welfare as

$$W(\{c_t^F, c_t(s^t)\}) = \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t c_t^F + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \log\left(c_t(s^t)\right) - d(\ell_t(s^t))\pi_t(s^t|s_0)\bar{\mu}(s_0),$$

where $\bar{\mu}^F$, $\bar{\mu}(e)$ and $\bar{\mu}(u)$ are the welfare weights of firm owners, initially employed and initially unemployed workers and $\pi_t(s^t|s_0)$ denotes the share of workers with history $s^t = (s_0, s_1, \ldots, s_t)$ in period t.

Definition 3 (Optimal allocation)**.** *An optimal allocation for a given sequence of productivity is a collection of sequences of aggregate consumption, capital, employment, search effort and labor market tightness and of individual consumption and search effort which solves the planner problem*

$$\begin{aligned} \max_{\{C_t, N_{t+1}, K_{t+1}, L_t^u, \theta_t, c_t^F, c_t(s^t), \ell_t(s^t)\}} \quad & W(\{c_t^F, c_t(s^t)\}) \\ \text{s.t. } & C_t + K_{t+1} + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t + \kappa_t \theta_t L_t^u \\ & = z_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta_k) K_t \\ & N_{t+1} = (1-\rho) N_t + q_t^v(\theta_t)\theta_t L_t^u \\ & C_t = \mu c_t^F + \sum_{s^t} c_t(s^t)\pi_t(s^t) \\ & L_t^u = \sum_{s^t | s_t = u} \ell_t(s^t)\pi_t(s^t) \\ \text{given } & K_0,\ N_0. \end{aligned} \tag{41}$$

The planner takes the sequence of productivity as given. In other words, the sequence of public investment and thereby productivity has already been decided, and the planner now faces the problem of allocating the remaining resources.[18] The first constraint in the planner problem is the aggregate resource constraint. The right-hand side are total available resources consisting of output and capital after depreciation which can be spent on consumption, investment in next period's

18. The costs of public investment could be added to the resource constraint without changing the results that follow. This is because firm owners have linear utility.

capital, and vacancy creation. The second constraint is the law of motion for employment. The planner can increase employment in the next period in two ways. First, the planner can raise tightness θ_t which comes at a resource cost according to the term $\kappa_t \theta_t L^u_t$ in the resource constraint since more vacancies have to be created for a constant level of aggregate search effort. Second, employment can be increased by raising aggregate search effort L^u_t with comes at a utility cost since effort enters the utility function, but there are also resource costs since more vacancies have to be created if tightness is to be held constant. The last two constraints of the planner problem state that individual consumption must add up to aggregate consumption and individual search effort $\ell_t(s^t)$ has to be consistent with aggregate search effort L^u_t.

The next propositions characterize the optimal allocation more closely.

Proposition 6 (Optimal allocation of capital)**.** *The optimal allocation of capital satisfies*

$$1+\phi\left(\frac{K_{t+1}}{K_t}-1\right) \\ =\beta\left(1+\alpha z_{t+1}k_{t+1}^{\alpha-1}-\delta_k+\frac{\phi}{2}\left(\left(\frac{K_{t+2}}{K_{t+1}}\right)^2-1\right)-\frac{\partial \kappa_{t+1}}{\partial K_{t+1}}\theta_{t+1}L^u_{t+1}\right).$$

Proof. The result follows immediately from the first-order conditions for consumption and capital associated with (41). □

If vacancy posting costs do not depend on capital, it holds that $\frac{\partial \kappa_{t+1}}{\partial K_{t+1}}=0$ and the optimal path for the aggregate capital stock coincides with the equilibrium allocation. However, if vacancy posting costs depend on the aggregate capital stock, for example because they are proportional to labor productivity as would be needed for balanced growth, then the aggregate capital stock is too high in equilibrium because existing firms who rent capital do not take into account that more capital per match makes it more expensive for new firms to post a vacancy.

Next, I characterize the sequence of optimal tightness. It will depend on the elasticity of the vacancy filling probability with respect to tightness which I denote as $\eta \equiv -\frac{m'(\theta_t)\theta_t}{q^v_t(\theta_t)}$.

Proposition 7 (Optimal tightness with fixed search effort)**.** *Suppose individual search effort is fixed at $\ell_t(s^t) = 1$ and $d(1,u) = d(1,e)$, then optimal tightness satisfies*

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta \Bigg\{ (1-\alpha) z_{t+1} k_{t+1}^{\alpha} - \eta \left[(1-\alpha) z_{t+1} k_{t+1}^{\alpha} + \kappa_{t+1}\theta_{t+1} \right] + (1-\rho) \frac{\kappa_{t+1}}{q_{t+1}^v(\theta_{t+1})} \Bigg\}.$$

Comparison with the equilibrium condition (7) shows that without search effort, the equilibrium is constrained efficient if the wage is

$$w_t = \eta \left[(1-\alpha) z_t k_t^{\alpha} + \kappa_t \theta_t \right] \tag{42}$$

This is the standard condition for efficiency in the DMP model.

Proposition 8 (Optimal tightness)**.** *Suppose that the welfare weights of initially unemployed and employed workers are equal to their population shares, $\bar{\mu}(s) = \pi_0(s)$, then optimal tightness satisfies*

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta \Bigg\{ (1-\alpha) z_{t+1} k_{t+1}^{\alpha} - \eta \left[(1-\alpha) z_{t+1} k_{t+1}^{\alpha} + \kappa_{t+1}\theta_{t+1}\ell_{t+1}(u) \right] + (1-\eta) \frac{\mu}{\bar{\mu}^F} \left(d(\ell_{t+1}(u), u) - d(0,e) \right) + (1-\rho) \frac{\kappa_{t+1}}{q_{t+1}^v(\theta_{t+1})} \Bigg\},$$

where the optimal level of individual search effort solves

$$d'(\ell_t(u), u) = \frac{\bar{\mu}^F}{\mu} \kappa_t \theta_t \frac{1}{1+\eta}.$$

In this case, the constrained-efficient allocation is implemented if the wage amounts to

$$w_t = \eta \left[(1-\alpha) z_t k_t^{\alpha} + \kappa_t \theta_t \ell_t(u) \right] - (1-\eta) \frac{\mu}{\bar{\mu}^F} \left(d(\ell_t(u), u) - d(0,e) \right). \tag{43}$$

The differences to the optimal wage in the case without effort given by equation (42) are intuitive. First, the term $\kappa_t \theta_t$ is multiplied by individual search effort $\ell_t(u)$. To see why, suppose optimal search effort increases. Then, firms find it easier to fill a vacancy and expand vacancy creation. To prevent an inefficiently high vacancy creation, the wage has to be higher to discourage vacancy creation. Second, the additional summand in (43) takes into account the difference in disutility of effort between employed and unemployed. If the disutility is higher for unemployed, a lower level of unemployment is desirable which is implemented through a lower wage leading to a higher level of labor market tightness.

B.3 No-trade equilibrium and interest rate

I want to show that hand-to-mouth behavior can be the equilibrium outcome in an extended model in which households can save in a risk-free bond a_t at rate r_t, but are borrowing constrained. Consider the following generalization of the household problems described in the main text. Workers are excluded from participation in the equity and capital market where firm owners trade shares and rent out capital. A worker's budget constraint is

$$c_t(s_t) \leq (1-\tau_t)w_t \mathbb{1}_{s_t=e} + b_t \mathbb{1}_{s_t=u}(1+r_t)a_t - a_{t+1} \tag{44}$$

with borrowing constraint $a_{t+1} \leq 0$. Workers choose private consumption, effort, and bond holdings $\{c_t(s_t), \ell_t(s_t), a_{t+1}(s_t)\}_{t=0}^{\infty}$ to maximize expected lifetime utility subject to the budget constraint (44) and the borrowing limit $a_{t+1} \geq 0$. Observe that the wage depends on asset holdings a_t since it is determined by Nash bargaining and worker are risk averse such that their surplus from employment depends on their asset holdings. In other words, workers are in a better bargaining position if they hold more assets since they will be able to sustain a higher level of consumption during unemployment. See Krusell et al. (2010) for a more extensive discussion of this mechanism.

Firm owners also have access to the bond market where they can trade bonds with workers and with each other. I assume that all firm owners have the same endowments in period $t = 0$. Then, since the labor market is the only source of idiosyncratic risk and firm owners do not participate in the labor market, they are identical at all times. The representative firm owner chooses a sequence of consumption, bond holdings, investment, and capital $\{c_t^F, a_{t+1}^F, i_t^F, k_{t+1}^F\}_{t=0}^\infty$ to maximize expected lifetime utility given an initial endowment of bonds and shares (a_0^F, x_0^F)

$$\max_{\{c_t^F, a_{t+1}^F, i_t^F, k_{t+1}^F\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^F)$$

$$\begin{aligned} \text{s.t. } c_t^F \leq& (1+r_t)a_t^F + \Pi_t^F - a_{t+1}^F + r_t^k k_t^F - T_t^F - \frac{\phi}{2}\left(\frac{i_t^F}{k_t^F} - \delta_k\right)^2 k_t^F - i_t^F, \\ k_{t+1}^F =& (1-\delta_k)k_t^F + i_t^F, \\ a_{t+1}^F \geq& 0, \quad k_{t+1}^F \geq 0. \end{aligned}$$

No-trade equilibrium Since the gross supply of the bond is zero and households cannot borrow, it must hold in equilibrium that households do not save, $a_t = 0$. This requires that the interest rate is low enough.

Proposition 9. *Consider the extended model described above. In equilibrium, $a_t = 0$ and it holds for the equilibrium interest rate*

$$\begin{aligned} 1 + r_{t+1} \leq& \frac{1}{\beta}\Bigg[\pi_t^{e|e} \frac{(1-\tau_t)w_t}{(1-\tau_{t+1})w_{t+1}} \\ & \times \left(1 + (1-\gamma)(1-\psi)\frac{\frac{w_{t+1}^N}{b_{t+1}} - 1 - (J_{t+1}(e) - J_{t+1}(u))}{1 + (1-\psi)(J_{t+1}(e) - J_{t+1}(u))}\right) \\ & + \pi_t^{u|e}\frac{(1-\tau_t)w_t}{b_{t+1}}\Bigg]^{-1} \end{aligned} \tag{45}$$

Proof. Consider an employed worker in period t. The choice $a_{t+1} = 0$ is optimal only if

$$\begin{aligned}\frac{1}{(1-\tau_t)w_t} \geq & \beta\left[\pi_t^{e|e}\frac{1}{(1-\tau_{t+1})w_{t+1}}\left(1+r_{t+1}+\frac{\partial w_{t+1}(a_{t+1})}{\partial a_{t+1}}(1+\tau_{t+1})\right)\right. \\ & \left. + \pi_t^{u|e}\frac{1+r_{t+1}}{b_{t+1}}\right].\end{aligned} \tag{46}$$

The derivative of the wage with respect to asset holdings, $\frac{\partial w_{t+1}}{\partial a_{t+1}}$, shows up because savings raise wages as they improve workers' bargaining position (Krusell et al. 2010). I now characterize this effect of savings on the wage. The Nash bargained wage is

$$w_t^N = \arg\max_w \psi \log\left(J_t^e(a,w) - J_t^u(a)\right) + (1-\psi)\log\left(J_t^F(w)\right),$$

where

$$\begin{aligned}J_t^e(a,w) = & \max_{a',\ell} \log\big(w(1-\tau_t) + (1+r_t)a - a'\big) \\ & + \beta\left[\pi_t^{e|e}(\ell)J_{t+1}^e(a',w') + \pi_t^{u|e}(\ell)J_{t+1}^u(a')\right]\end{aligned}$$

and

$$\begin{aligned}J_t^u(a) = & \max_{a',\ell} \log\big(b_t + (1+r_t)a - a'\big) \\ & + \beta\left[\pi_t^{e|u}(\ell)J_{t+1}^e(a',w') + \pi_t^{u|u}(\ell)J_{t+1}^u(a')\right]\end{aligned}$$

are the worker value functions of the extended model with savings.

The following first order condition implicitly defines the Nash wage

$$\begin{aligned}0 = F(w_t^N, a) \equiv & (1-\psi)\left(J_t^e(a,w_t^N) - J_t^u(a)\right)\frac{w_t^N(1-\tau) + (1+r_t)a - a'}{1-\tau_t} \\ & - \psi\left(J_t^F(w_t^N)\right).\end{aligned}$$

Its derivates with respect to the Nash-wage and asset are

$$\begin{aligned}\frac{\partial F(w_t^N, a)}{\partial w_t^N} =&(1-\psi)\Bigg[\left(\frac{\partial J_t^e(a, w_t^N)}{\partial w_t^N} - \frac{\partial J_t^u(a)}{\partial w_t^N}\right) \\ &\times \frac{w_t^N(1-\tau) + (1+r_t)a - a'}{1-\tau} \\ &+ J_t^e(a, w_t^N) - J_t^u(a)\Bigg] - \psi \frac{\partial J_t^F(w_t^N)}{\partial w_t^N}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial F(w_t^N, a)}{\partial a} =&(1-\psi)\Bigg[\left(\frac{\partial J^e(w_t^N, a)}{\partial a} - \frac{\partial J^u(a)}{\partial a}\right) \\ &\times \frac{w_t^N(1-\tau) + (1+r_t)a - a'}{1-\tau} \\ &+ (J_t^e(w_t^N, a) - J_t^u(a))\frac{1+r_t}{1-\tau}\Bigg].\end{aligned}$$

Using the implicit function theorem, it follows that

$$\frac{\partial w_t}{\partial a} = -(1-\gamma)\frac{(1-\psi)(1+r)}{1-\tau}\frac{1-(1-\tau_t)\frac{w_t^N}{b_t} + (J_t^e(w_t^N, a) - J_t^u(a))}{1+(1-\psi)(J_t^e(w_t^N, a) - J_t^u(a))}.$$

Substituting this into (46) with $a = 0$ and $w_t^N = w_t$ yields

$$\begin{aligned}\frac{1}{(1-\tau_t)w_t} \geq&\beta\,(1+r_{t+1})\Bigg[\pi_t^{e|e}(\ell_t)\frac{1}{(1-\tau_{t+1})w_{t+1}} \\ &\times\left(1+(1-\gamma)(1-\psi)\frac{\frac{w_{t+1}^N}{b_{t+1}} - 1 - (J_{t+1}(e) - J_{t+1}(u))}{1+(1-\psi)(J_{t+1}(e) - J_{t+1}(u))}\right) \\ &+ \pi_t^{u|e}(\ell_t)\frac{1}{b_{t+1}}\Bigg].\end{aligned}$$

Solving for the interest rate gives,

$$
\begin{aligned}
1+r_{t+1} \leq & \frac{1}{\beta}\left[\pi_t^{e|e} \frac{(1-\tau_t) w_t}{(1-\tau_{t+1}) w_{t+1}}\right. \\
& \times\left(1+(1-\gamma)(1-\psi) \frac{\frac{w_{t+1}^N}{b_{t+1}}-1-\left(J_{t+1}(e)-J_{t+1}(u)\right)}{1+(1-\psi)\left(J_{t+1}(e)-J_{t+1}(u)\right)}\right) \\
& \left.+\pi_t^{u|e} \frac{(1-\tau_t) w_t}{b_{t+1}}\right]^{-1} .
\end{aligned}
$$

Here, ℓ_t is the effort choice of unemployed workers in the equilibrium of the main text where saving is ruled out. For this interest rate, the necessary condition for optimality of $a_{t+1} = 0$ is satisfied. □

For the calibration of the model, I assume that equation (45) holds with equality, i.e, the equilibrium interest rate is

$$
\begin{aligned}
1+r_{t+1} = & \frac{1}{\beta}\left[\pi_t^{e|e} \frac{(1-\tau_t) w_t}{(1-\tau_{t+1}) w_{t+1}}\right. \\
& \times\left(1+(1-\gamma)(1-\psi) \frac{\frac{w_{t+1}^N}{b_{t+1}}-1-\left(J_{t+1}(e)-J_{t+1}(u)\right)}{1+(1-\psi)\left(J_{t+1}(e)-J_{t+1}(u)\right)}\right) \\
& \left.+\pi_t^{u|e} \frac{(1-\tau_t) w_t}{b_{t+1}}\right]^{-1} .
\end{aligned} \tag{47}
$$

This choice can be justified as the equilibrium interest rate in the limit as the supply of bonds goes to zero, which Werning (2015) labels the case of vanishing liquidity.

Condition (45) is only a necessary condition. It may not be sufficient for two reasons. First, it ensures that employed workers do not save, but unemployed workers might still do so if the job finding probability is high relative to the separation rate. In this case, unemployed workers have a stronger incentive to save than employed workers. We can obtain a condition similar to (45), that is necessary to rule out saving of unemployed workers. Second, because of the endogenous effort choice, households' expected utility is not necessarily concave in effort and assets. Starting from zero savings, a simultaneous increase in savings and decrease in effort may raise expected lifetime utility. Hence, I numerically verify that $a_t = 0$ is indeed an optimal choice for all households if the interest rate is given by (47).

B.4 News shock

The preceding discussion has highlighted the role of expectation about future productivity for the employment effect of public investment. The importance of expected future productivity can also be seen when comparing the employment effect of public investment to the change in employment that would result from a permanent change in productivity, defined as follows.

Definition 4 (Employment effect of (future) productivity)**.** *Denote by $N_t(\mathcal{Y}_0, z_0, z_1, \dots)$ employment in period t in an equilibrium with initial conditions $\mathcal{Y}_0 = (N_0, w_0, K_0)$ and productivity sequence $\mathcal{Z} = (z_t)_{t=0}^{\infty}$. Consider a permanent increase in productivity in period T. The employment effect in t is defined as*

$$M_t^z(T, \mathcal{Y}_0, \mathcal{Z}) = \frac{\partial N_t(\mathcal{Y}_0, \dots, z_{T-1}, xz_T, xz_{T+1}, \dots)}{\partial x}|_{x=1}.$$

I get the following result

Proposition 10. *If the economy is in its steady state initially, then*

$$M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = \frac{\vartheta}{1 - \beta(1 - \delta_G)(1 - \rho)} \frac{\delta_G}{I^G} M_t^z(T, \mathcal{Y}_0, \mathcal{Z}).$$

The employment effect of public investment is proportional to the employment change in response to a permanent change in future productivity where the factor of proportionality depends on the elasticity of productivity with respect to public investment. For private agents, the announcement of the public investment expansion constitutes a news shock about productivity and, up to a constant factor, induces the same employment response.

Proof. Consider the productivity sequence $(z_k)_{k=0}^{\infty}$ with $z_k = z$ for $k < t$ and $z = xz$ for $k \geq T$. The wage in period s is

$$w_s = \begin{cases} \gamma^s w_0 + (1-\gamma)\omega a^\alpha z^{\frac{1}{1-\alpha}} \frac{\gamma^s - 1}{\gamma - 1}, \text{ if } s < T \\ \gamma^s w_0 + (1-\gamma)\omega a^\alpha z^{\frac{1}{1-\alpha}} \left(\frac{\gamma^s - \gamma^{s-T+1}}{\gamma - 1} + x^{\frac{1}{1-\alpha}} \frac{\gamma^{s-T+1} - 1}{\gamma - 1} \right), \text{ if } s \geq T \end{cases}$$

and for $k < T$

$$\begin{aligned} \pi_k^{e|u} =& \zeta^{\frac{1}{\eta}} (1-\rho)^{\frac{1-\eta}{\eta}} \left(z^{\frac{1}{1-\alpha}} \kappa a^\alpha \right)^{\frac{\eta-1}{\eta}} \\ & \times \Bigg[\sum_{s=k}^{T-1} (\beta(1-\rho))^{s-k} (1-\alpha) a^\alpha z^{\frac{1}{1-\alpha}} \\ & + \sum_{s=T}^{\infty} x^{\frac{1}{1-\alpha}} (\beta(1-\rho))^{s-k} (1-\alpha) a^\alpha z^{\frac{1}{1-\alpha}} \\ & - \sum_{s=k}^{\infty} (\beta(1-\rho))^{s-k} \gamma^s w_0 + \sum_{s=k}^{T-1} (\beta(1-\rho))^{s-k} \omega a^\alpha z^{\frac{1}{1-\alpha}} (\gamma^s - 1) \\ & + \sum_{s=T}^{\infty} (\beta(1-\rho))^{s-k} \omega a^\alpha z^{\frac{1}{1-\alpha}} (\gamma^s - \gamma^{s-T+1}) \\ & + \sum_{s=T}^{\infty} (\beta(1-\rho))^{s-k} \omega a^\alpha z^{\frac{1}{1-\alpha}} x^{\frac{1}{1-\alpha}} (\gamma^{s-T+1} - 1) \Bigg]^{\frac{1-\eta}{\eta}} \end{aligned}$$

which can be simplified to

$$\begin{aligned} \pi_k^{e|u} =& \zeta^{\frac{1}{\eta}} (1-\rho)^{\frac{1-\eta}{\eta}} \left(z^{\frac{1}{1-\alpha}} \kappa a^\alpha \right)^{\frac{\eta-1}{\eta}} \\ & \Bigg\{ \frac{(1-\alpha-\omega) a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} \left[1 + (\beta(1-\rho))^{T-k} (x^{\frac{1}{1-\alpha}} - 1) \right] \\ & + \gamma \frac{\omega a^\alpha z^{\frac{1}{1-\alpha}}}{1-\gamma\beta(1-\rho)} (\beta(1-\rho))^{T-k} (x^{\frac{1}{1-\alpha}} - 1) \\ & - \frac{\gamma^k}{1-\gamma\beta(1-\rho)} (w_0 - \omega a^\alpha z^{\frac{1}{1-\alpha}}) \Bigg\}^{\frac{1-\eta}{\eta}} \end{aligned}$$

I have that for $k < T$

$$\begin{aligned}\frac{\partial \pi_k^{e|u}}{\partial x}|_{x=1} =& \pi_k^{e|u} \frac{1-\eta}{\eta} \frac{1}{1-\alpha} \left\{ \frac{(1-\alpha-\omega) a^{\alpha} z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} \right. \\ & \left. - \frac{\gamma^k}{1-\gamma\beta(1-\rho)} (w_0 - \omega a^{\alpha} z^{\frac{1}{1-\alpha}}) \right\}^{-1} (\beta(1-\rho))^{T-k} \\ & \left(\frac{(1-\alpha-\omega) a^{\alpha} z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} + \gamma \frac{\omega a^{\alpha} z^{\frac{1}{1-\alpha}}}{1-\gamma\beta(1-\rho)} \right),\end{aligned}$$

If the wage in period 0 is at its steady state value $w_0 = \omega a^{\alpha} z^{\frac{1}{1-\alpha}}$, I have for $k < T$

$$\begin{aligned}\frac{\partial \pi_k^{e|u}}{\partial x}|_{x=1} =& (\beta(1-\rho))^{T-k} \pi_k^{e|u} \frac{1-\eta}{\eta} \frac{1}{1-\alpha} \\ & \times \left(1 + \frac{\omega\gamma}{1-\omega-\alpha} \frac{1-\beta(1-\rho)}{1-\gamma\beta(1-\rho)} \right) > 0.\end{aligned}$$

Note that $1-\omega-\alpha > 0$ if $\pi_k^{e|u} > 0$. The short-run employment effect is

$$\begin{aligned}M_t^z(T, \mathcal{Y}_0, \mathcal{Z}) =& (1-N_0) \frac{\partial \pi_0^{e|u}}{\partial x} = (1-N_0)(\beta(1-\rho))^T \pi_0^{e|u} \frac{1-\eta}{\eta} \frac{1}{1-\alpha} \\ & \times \left(1 + \frac{\omega\gamma}{1-\gamma\beta(1-\rho)} \frac{1-\beta(1-\rho)}{1-\omega-\alpha} \right)\end{aligned}$$

If the economy is at the steady state initially, then the employment effect is

$$\begin{aligned}M_t^z(T, \mathcal{Y}_0, \mathcal{Z}) &= \sum_{k=0}^{t-1} (1-\rho-\pi^{e|u})^{t-k-1} (1-N) \frac{\partial \pi_k^{e|u}}{\partial x} \\ &= (\beta(1-\rho))^{T+1-t} (1-N) \pi^{e|u} \frac{1}{1-\alpha} \frac{1-\eta}{\eta} \\ &\times \frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^t}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))} \\ &\times \left(1 + \frac{\omega\gamma}{1-\gamma\beta(1-\rho)} \frac{1-\beta(1-\rho)}{1-\omega-\alpha} \right).\end{aligned}$$

The result then follows from the formula for the employment multiplier of public investment (22). □

C Additional Quantitative Results

Table 6: Business cycle moments with cross-correlations.

	U	Y	Inv.	Wages	Lab. prod.	z
Standard dev.	0.081 (0.101)	0.017 (0.015)	0.090 (0.065)	0.008 (0.010)	0.011 (0.012)	0.012 (0.012)
Autocorr.	0.848 (0.943)	0.846 (0.845)	0.248 (0.821)	0.947 (0.744)	0.789 (0.761)	0.791 (0.797)
Corr. with ...						
U	1.000 (1.000)	-0.931 (-0.858)	-0.674 (-0.800)	-0.594 (-0.300)	-0.836 (0.127)	-0.882 (-0.345)
Y	—	1.000 (1.000)	0.761 (0.878)	0.703 (0.553)	0.973 (0.624)	0.983 (0.788)
inv.	—	—	1.000 (1.000)	0.276 (0.455)	0.760 (0.595)	0.808 (0.756)
wages	—	—	—	1.000 (1.000)	0.717 (0.012)	0.618 (0.012)
lab. prod.	—	—	—	—	1.000 (1.000)	0.985 (0.872)
z	—	—	—	—	—	1.000 (1.000)

Data moments in parentheses. Variables are relative deviations from the HP trend with smoothing parameters 1,600. We use quarterly data from 1951q1 to 2019q4.

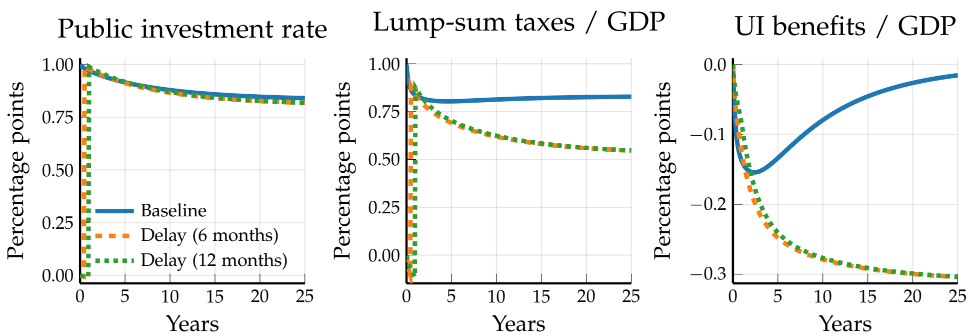


Figure 11: The fiscal response to the public investment expansion.

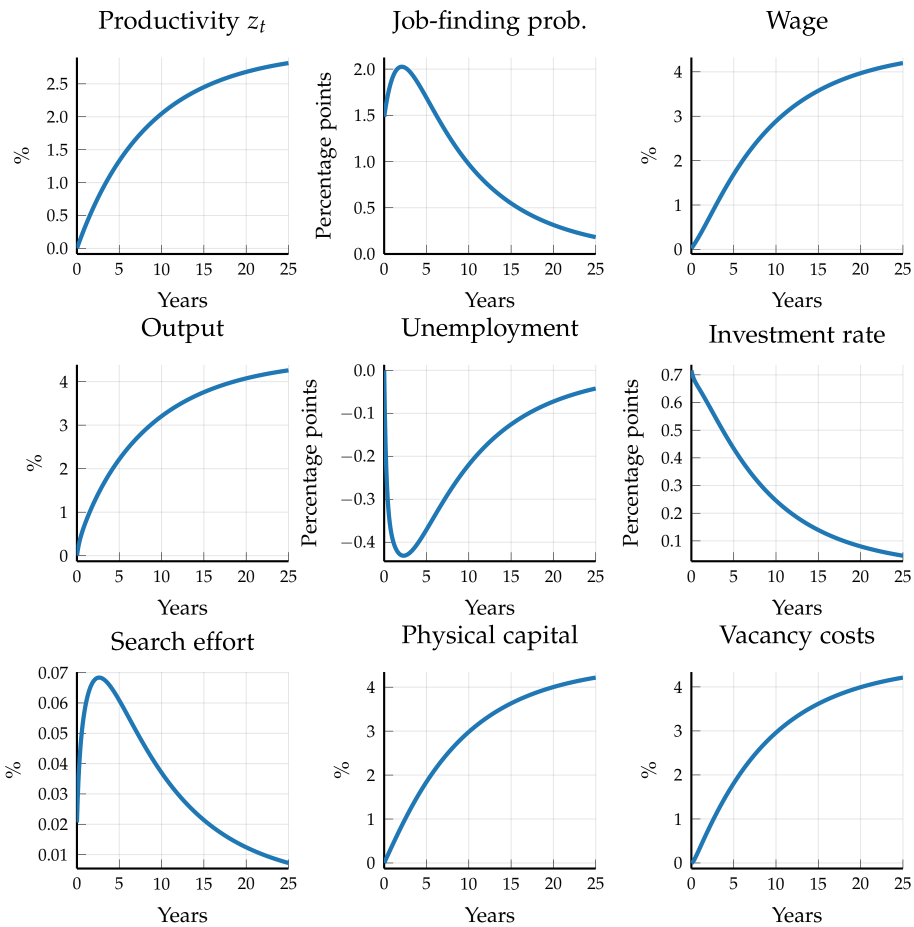


Figure 12: Long-run responses to a government investment program.

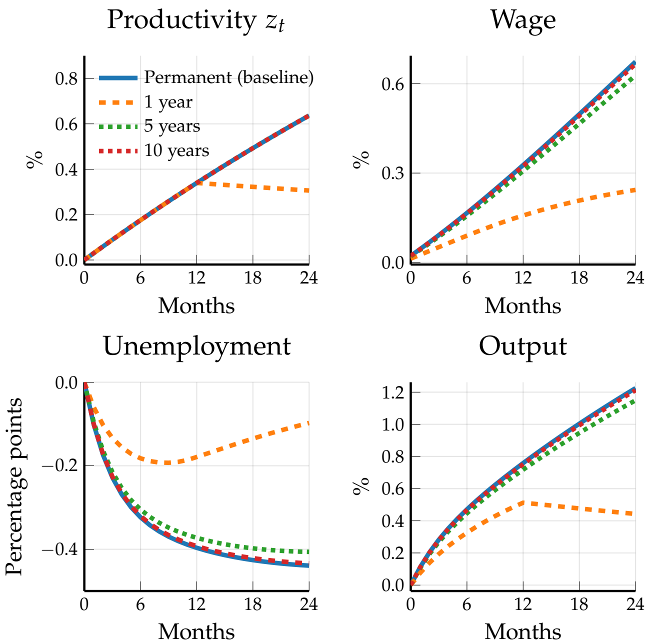


Figure 13: Responses to transitory expansion in public investment.

Table 7: Output multipliers of public investment at different horizons for different scenarios.

		1 year	2 years	3 years	Long run
Peak	baseline	0.71	1.18	1.57	4.52
	6 months delay	0.47	0.98	1.40	4.52
	12 months delay	0.24	0.76	1.22	4.52
	recession	0.79	1.23	1.61	4.52
	labor tax financed	0.54	0.98	1.36	4.42
Cumulative	baseline	0.41	0.69	0.93	4.52
	6 months delay	0.45	0.66	0.88	4.52
	12 months delay	—	0.66	0.84	4.52
	recession	0.48	0.76	0.98	4.52
	labor tax financed	0.30	0.54	0.76	4.42

Notes: Peak multiplier: maximum change in output over change in public investment over the respective horizon H, $\max_{h\leq H} \frac{\Delta Y_h}{\Delta I^G_h}$. Cumulative multiplier: cumulative change in output over horizon H over cumulative change in public investment over the same horizon, $\frac{\sum_{h\leq H} \Delta Y_h}{\sum_{h\leq H} \Delta I^G_h}$.

C.1 State dependence

For the case without an expansion in public investment, Figure 14 shows the evolution of unemployment, labor market tightness and wages starting from the two different initial conditions (boom and recession) described in the main text.

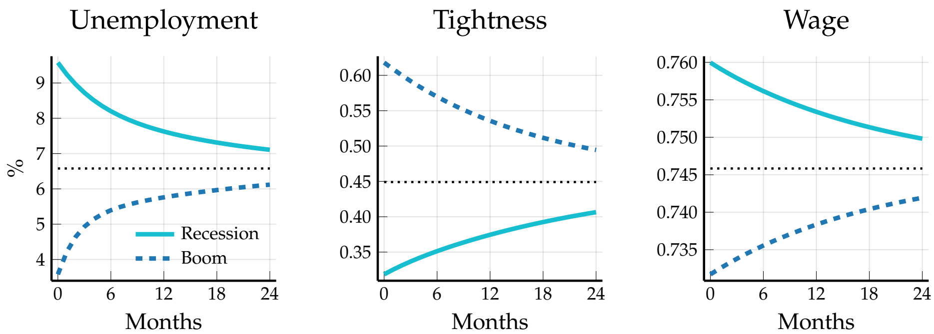


Figure 14: Unemployment, labor market tightness and wages in recession and boom.
Notes: The black dotted line denotes the steady state. Unemployment in percent, wages in units of the consumption good.

In the main text, I study the state dependence of the employment effect of public investment considering a recession that results from a joint positive shock to the separation rate and the wage level (and vice-versa for a boom). Here, I alternatively consider a recession due to a negative shock to productivity of one standard deviation,

$$\log A_0 = -0.0056$$

and accordingly, for a boom

$$\log A_0 = +0.0056,$$

after which productivity A_t evolves according to (29).

Figure 15 show the response of TFP, unemployment, labor market tightness and wages.

Qualitatively, I obtain the same result as in the main text—the employment effect of public investment is larger in a recession.

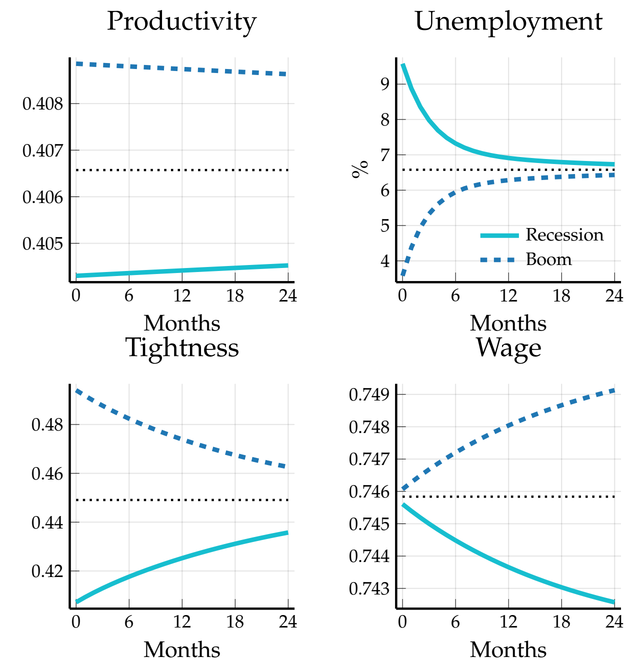


Figure 15: Responses to productivity shocks.

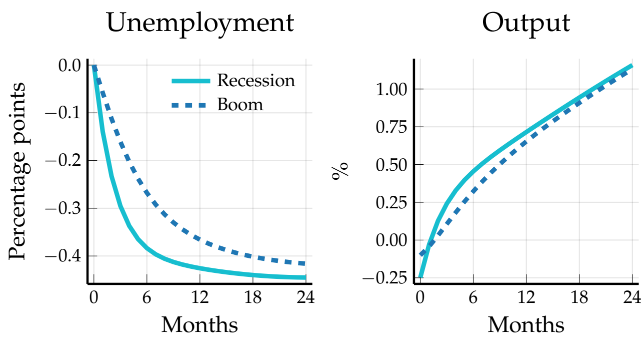


Figure 16: Responses of unemployment and output to permanent expansion in public investment in recession and boom.
Notes: Shown are the deviations from the paths without an expansion in public investment (see Figure 15)

C.2 Alternative parameterizations

For the baseline calibration I have chosen the bargaining power of workers ψ such that the labor share is 64% as in the data. Alternatively, I could require that the bargaining power is such that vacancy creation is efficient in the steady state, i.e., the wage is given by (43). Note that the right term in (43) is zero in the steady state given our calibration strategy. The employment and wage response for a re-calibration of the model that requires workers bargaining power to implement efficient vacancy creation in steady state is shown by the dashed red line in Figure 17.

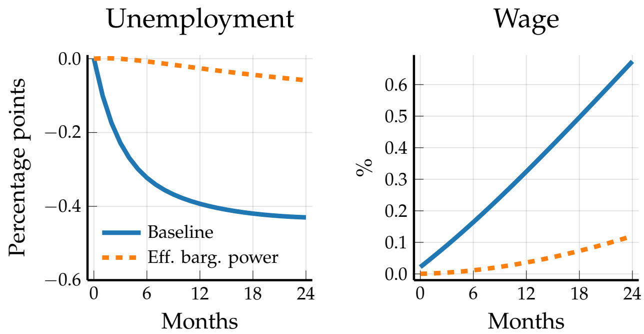


Figure 17: Response if bargaining power implements efficient vacancy creation in steady state.
Notes: Dashed red line: response of unemployment for calibration where workers' bargaining power is chosen to implement efficient level of vacancy creation in steady state.

Our baseline specification assumes that posting costs are proportional to labor productivity. The dotted red line in Figure 18 shows the short-run response of unemployment and wages if posting costs are constant instead. The dashed orange line shows the responses when capital adjustment costs are zero. The dashed green line shows the responses when capital adjustment costs are infinite, i.e., the private capital stock is constant.

Figure 19 varies the degree of wage stickiness.

Figure 18: Responses without capital adjustment costs and with constant vacancy posting costs.
Notes: Dashed orange line: no capital adjustment costs. Dashed green line: infinite capital adjustment costs. Dotted red line: constant posting costs.

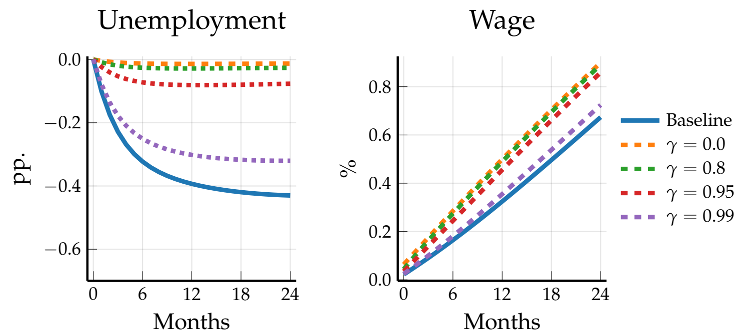


Figure 19: Responses for different degrees of wage stickiness.

References

Den Haan, Wouter J., and Georg Kaltenbrunner. 2009. "Anticipated Growth and Business Cycles in Matching Models." *Journal of Monetary Economics* 56 (3): 309–327.

Hosios, Arthur J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *The Review of Economic Studies* 57 (2): 279–298.

Krusell, Per, Toshihiko Mukoyama, and Ayegül ahin. 2010. "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations." *The Review of Economic Studies* 77 (4): 1477–1507.

Shimer, Robert. 2012. "Reassessing the Ins and Outs of Unemployment." *Review of Economic Dynamics* 15 (2): 127–148.

Werning, Iván. 2015. *Incomplete Markets and Aggregate Demand.* National Bureau of Economic Research Working Paper 21448.